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Signals and Interference in FM Reception: I. Deterministic Models—The "Instantaneous" Approach, with Undistorted Inputs

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Signals and Interference in FM Reception: I. Deterministic Models—The "Instantaneous" Approach, with Undistorted Inputs

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I

PREFACE

This Report is the first (i.e. Part I) in a particular series of ongoing studies of the general electromagnetic interference (EMI) environment specifically devoted to the performance of common classes of reception in such environments. Typically, one is concerned here with more or less conventional FM and AM receivers, for both analogue and digitalized signals. And, typically, the EMI environment is often composed of similar interfering signals. One principal concern is the performance of FM receivers, when the interference is not the familiar gaussian or normal noise of conventional noise sources, but rather the highly structured, non-gaussian noise produced by undesired signal inputs, whether man-made or not, or "intelligent" (i.e., message-bearing) or not.

Although the pursuit of optimality in reception is always necessary, if only to establish theoretical bounds on possible performance and to indicate optimal signal processing algorithms which can be approximated to varying degrees in practice, the evaluation of the performance of commonly-used, sub-optimum systems is equally necessary, since such systems are relatively ubiquitous currently and are likely to remain so to some extent for an indefinite future period. The recent development of analytically tractable, nongaussian interference models, based on statistical-physical mechanisms has greatly assisted the treatment of the optimality problem as well as permitting comparisons with a number of common, comparatively simple sub-obtimum receivers (e.g., digital signals in FSK, ϕSK, etc.), work which is continuing in a parallel effort. The general aim of the present Report, and its successors, is to consider the performance of more complex, comparatively non-ideal receivers, in particular the FM receiver, including specifically the various nonlinearities which make such systems so challenging to analyze.

In a broader sense, the material developed here is designed to assist the quantitative treatment of signals and interference in various nonlinear reception systems, generally, not only in regard to the specific topic of FM.

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SIGNALS AND INTERFERENCE IN FM RECEPTION: I. DETERMINISTIC MODELS - THE "INSTANTANEOUS" APPROACH, WITH UNDISTORTED INPUTS

David Middleton* and A. D. Spaulding**

The purposes of this study (and subsequent efforts) are several: (1), to extend earlier models of the FM reception process, to include as much "realisim" - i.e., non-ideality of both the linear and nonlinear elements of the typical FM receiver - as possible, and still retain analytical and computational feasibility; (2), to examine explicit cases of interference produced by one or more deterministic signals; and (3), with such specific examples, both to provide insights into the distortion effects generated by the nonlinear interactions of the various (desired and undesired) signals in the receiver and to present the analytical framework of the instantaneous outputs required in any (subsequent) fully statistical treatment, where now the interference (e.g., "noise") is noticeably nongaussian. In addition, these deterministic models may also provide useful structures for simulation studies.

The instantaneous receiver outputs are obtained for the following receiver models, (A), and interference "scenarios", (B): for (A): (I) "superclipping: and an ideal discriminator; (II), no limiting and ideal discriminator; III, "superclipping" and a nonideal IV, no limiting and a nonideal discriminator. For (B), with each (A), we consider explicitly the cases of: (i), one cochannel interfering signal; (ii), one adjacent channel interferer, and (iii), M symmetrical interferers (M = 1, 5). Also included are the mean and mean-square outputs. All the above are obtained here for idealized (i.e. sufficiently wide-band) RF-IF receiver stages, which are essentially linear under this condition. The results are illustrated with cases for selected, typical parameters of the combination of the interference-receiver structure. For other combinations, the appropriate computer programs are included in the Appendix.

Key Words: FM reception, interference, receiver models, multiple interferers, baseband output waveforms

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1. INTRODUCTION

The problems of FM reception of signals in noise and signals in interference (i.e. other, similar, but undesired signals) are by now "classical": these problems have received continuing attention for over four decades, as the selected papers in Ref. 7 indicates. This is not at all surprising, for, both from the academic and practical viewpoints, these problems are not fully resolved and remain important and continuing challenges to the specialist in signal reception in particular and to those who would apply the results to spectrum management questions generally, as well as to other areas of telecommunications.

As usual, the core of the technical difficulties are the non-linear operations imposed by the receiver upon the combination of desired and undesired signals (as well as the inherent receiver noise). Even with receiver noise alone, and at most the desired signal, the analysis is quite involved, although largely tractable, as Rice,[8] Blachman,[9] and Middleton,[10], for example, have shown. The addition of interfering signals greatly complicates the analysis, because of the highly non-gaussian nature of such interference,[1-3] unlike receiver noise, which is, of course, normal (i.e., guassian). Another, inherent complication is the combination of non-linear operations embodied in a typical FM receiver: (1), the antenna aperture x RF x IF stages, essentially linear for AM signals, introduce nonlinear effects, essentially modulations of the carrier(s) by this (linear) front-end filter response; (2), the limiter, of course, further distorts the incoming wave, ideally to remove all amplitude variations, while (3); the discriminator non-ideally acts to convert an instantaneous frequency into an output voltage. Even in the semi-idealized cases treated here (Part I), where the front-end response is postulated to be sufficiently broad spectrally to introduce no distortions of the input, the significant nonlinearities of limiter and discriminator necessarily remain.

The principal aims of this present study are to extend earlier models of FM reception to include as much "realism", namely, nonidealized nonlinear elements, as possible while still retaining analytic and computational tractability, and to develop the analytic framework relating the instantaneous inputs and outputs, in the general case of arbitrary sets of interfering signals. From these, in turn, we can obtain various temporal averages of the output, as well as the (distorted) waveforms, in specific relationship to both the signal and receiver parameters from which one can determine various distorting effects of the interference upon the desired signal. (We note that only when the desired (modulating) signal is sinusoidal, i.e., contains but a single frequency component, can the amount of distortion produced by the interference (at other modulating frequencies), and the receiver, be specified unambiguously. This situation, of course, does not apply when the only undesirable "signals" are receiver noise, which is gaussian and therefore analytically and physically separable from the desired deterministic signal in the receiver output.)

Our treatment here is deterministic: the signal structures are treated as non-random components of the input. This is a necessary initial approach, not only for any subsequent statistical analysis, but also for the direct attack on specific interference scenarios involving one or more explicit waveforms. In addition to quantitative results, considerable insight may also be obtained into the qualitative effects of these nonlinear systems in varieties of input signals and interference.

In handling nonlinear problems of the above class we shall see (below) that a fully general approach is, perhaps somewhat surprisingly, much easier and analytically simpler than the attempt to analyze "simple" special cases ab initio. Subsequent redirection to special cases of interest is then made at once, without loss of key model structure or the introduction of implicit assumptions. It is also rather surprising to see how far this direct analysis can be carried in tractable form, before approximations, computational methods and possibly simulation of some of the analytic forms must be resorted to. In many instances (see also below) reasonable analytic solutions appear possible, at least for the instantaneous receiver output. The basic reason for the comparative simplicity and compactness of the general analytic forms obtained

stems from the direct approach of determining the instantaneous waveforms, rather than attempting harmonic (i.e. spectral) analyses at the outset or during the evaluation of the received signal models. (This accounts for the rather conspicuous absence of the ubiquitous Bessel functions which appear in most conventional analyses.[7])

The principal new elements of the present work are, accordingly: (1); the direct analytical development of the instantaneous input-output relationship when there is an arbitrary "scenario" of interfering signals; and (2), the numerical evaluation and comparison of the resulting waveforms, for various choices of signal and receiver parameters; [also, (3), the "software", by which arbitrary combinations of parameter values may be specifically calculated, as the need may arise.] In particular, this includes non-ideal discriminators; general interference, with possible combined AM and angle modulations; explicit results for instantaneous envelope (E) and frequency ($\dot{\theta}$) and explicit models (E,θ) for direct structural inputs for simulation at different levels of complexity and direct computational attack.

Finally, the paper is organized as follows: Section 2 below gives the formulation of the FM receiver as a nonlinear system employing nonideal elements. Our aim here is to obtain an explicit relation between the input (EM) wave, $V_{in}(t)$, entering the antenna aperture ⊗ RF ⊗ IF stages of the receiver, and the final, low-frequency output $E_o(t)$, representing the signal wave, which is then further processed by a human or automatic observer. Section 3 outlines various possible measures of the receiver output, which may provide useful and revealing insight into receiver performance, both on an instantaneous and average basis. Section 4 provides the general expresssion for the instantaneous frequency and envelope outputs of the receiver's front-end stages, for the multiple signal interference model, while in Sec. 5 various complexity levels of receiver model are identified and discussed, from the ideal to the fully "actual", non-ideal cases. Here the basic, desired quantities to be computed are briefly indicated, pertaining to the (instantaneous) analytic results obtained before. Section 6 then outlines the particular interference "scenarios" to be examined, at least analytically, and in some instances computationally as well. Section 7 gives an interpretation of the numerical results obtained, and Section 8 completes this paper with a brief discussion of the principal

results achieved. [An Appendix contains the computer programs which give the "software" required to perform the calculations.]

2. THE "INSTANTANEOUS" MODEL OF A NONIDEAL FM RECEIVER[10]

In Fig. 1, we sketch the diagram of a typical non-ideal FM receiver, indicating the various linear and non-linear elements. Our goal is to relate the input wave, $V_{in}(t)$, to the low-frequency output, $E_o(t)$.

2.1. The Input-Output Relations for the ARI Stages

We begin by considering the input and outputs $V_{in}(t)$, $V_o(t)$ of the linear front-end (ARI≡antenna aperture ⊗ RF ⊗ IF) stages of the receiver [cf. Fig. 1]. The input is usually narrow-band, though not necessarily spectrally narrower or "in-tune" with the ARI stages, which, however, are always narrow-band (about $f=f_o$). .We have[10]

$$V_{in}(t) = \mathrm{Re}\{[\alpha_{in}(t) - \beta_{in}(t)]e^{i\omega_c t}\} \text{ , or} \tag{2.1}$$

$$= \alpha_{in}(t)\cos\omega_c t + \beta_{in}(t)\sin\omega_c t \text{ ,} \tag{2.1a}$$

where α_{in}, β_{in} are in the (n.b.) "in-phase" and "out-of-phase" components of the n.b. input $V_{in}(t)$. The n.b. output of the ARI stages is found to be (Ref. 10, p. 637)

$$V_o(t) = \mathrm{Re}\left(\int_{-\infty}^{\infty}\{h_o(\tau)e^{-i\gamma_o(\tau)+i\omega_D(t-\tau)}\}_{ARI}\beta_o(t-\tau)e^{-i\phi(t-\tau)}d\tau\right)e^{i\omega_o t}, \tag{2.2}$$

$$\omega_D = \omega_c - \omega_o \text{ ,}$$

where $h_o e^{-i\gamma_o}$ is the "low-frequency" form of the n.b. ARI filter (cf. p. 98, Ref. 10), e.g.

$$h(t)_{ARI} = 2\mathrm{Re}\left\{h_o(t)_{ARI}e^{-i\gamma_o(t)_{ARI}+i\omega_o t}\right\} \text{ ,} \tag{2.3a}$$

$$\{h_o(t)e^{-i\gamma_o(t)}\}_{ARI} = \int_{-\infty}^{\infty}Y_o(i\omega')e^{i\omega' t}df', \ \omega'=2\pi f' \ ; \ \omega' = \omega-\omega_o, \tag{2.3b}$$

and where h_o, γ_o are (slowly-varying and) real quantities. Here ω_c is the "shifted" (by the "mixer", to current RF carrier, ω_c', to IF carrier ω_o)

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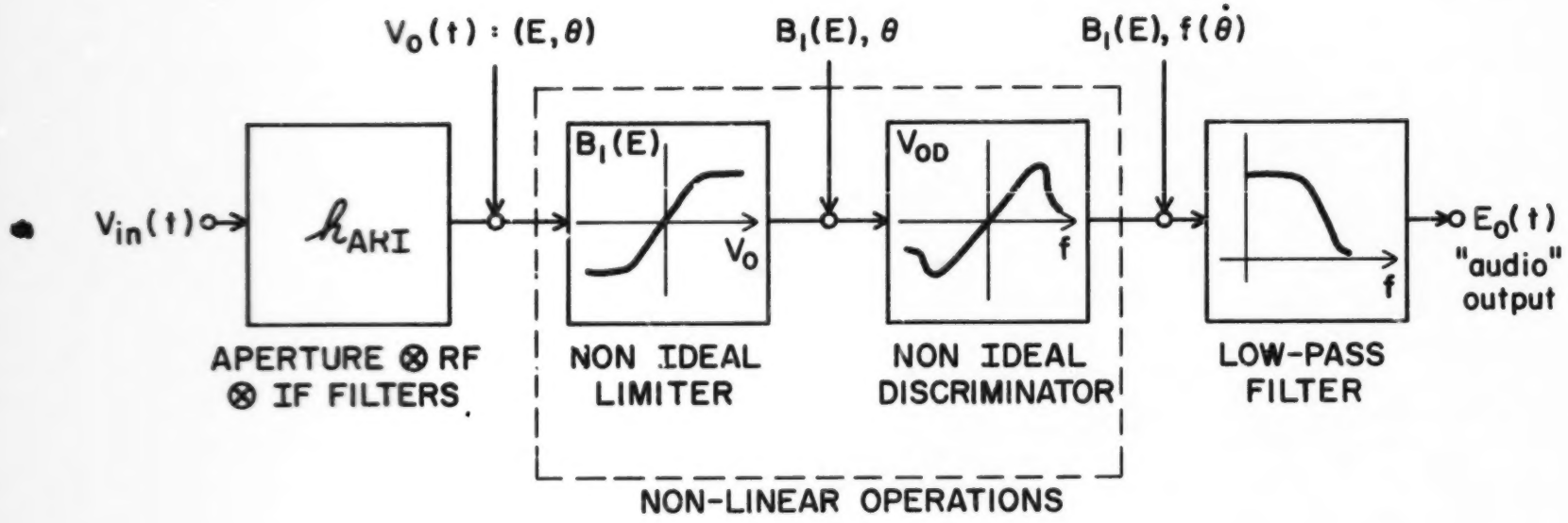

Figure 1. Operational schematic of a typical non-ideal (narrowband) FM receiver.

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input signal frequency, while ω_o ($=2\pi f_o$), as usual, is the IF (angular) center (or "carrier") frequency.

Similarly, we can express $V_o(t)$ as, cf. (2.1):

$$V_o(t) = \mathrm{Re}\{[\alpha_o(t)-i\beta_o(t)]e^{i\omega_c t}\}\ , \tag{2.4}$$

or, in terms of an instantaneous envelope and phase (E=) $A_o(t)$, ($\theta\equiv$) $\Psi_o(t)$ (real):

$$V_o(t) = \mathrm{Re}\left\{A_o(t)e^{i\omega_c t - i\Psi_o(t)}\right\}\ ,\quad \omega_c = \omega_D+\omega_o, \tag{2.5}$$

where

$$E(t) \equiv A_o(t) = \sqrt{\alpha_o^2+\beta_o^2}\quad (\geq 0);\quad \Psi_o(t) = \tan^{-1}(\beta_o/\alpha_o) \equiv \theta(t)\ , \tag{2.5a}$$

so that, comparing with (2.1), (2.2), we have

$$A_o(t)e^{i\omega_D t - i\Psi_o(t)} = e^{i\omega_D t}\int_{-\infty}^{\infty}[B_o(t-\tau)e^{-i\Phi_o(t-\tau)}][h_o(\tau)e^{-i\gamma_o(\tau)}]_{ARI}e^{-i\omega_D\tau}\,d\tau. \tag{2.6}$$

for the fundamental expression relating the n.b. input, output, and "n.b." portion of the ARI stages.

2.2. The Non-Ideal Limiter

This device [cf. Fig. 1] is described satisfactorily in Section 15.1-2 of Ref. 10. The output of the general (zero-memory) limiter for the first spectral zone, i.e. for the distinct spectral regions about f_o, is given by

$$B_1(E) = \frac{i}{\pi}\int_C f_L(i\xi)J_1(\xi E)\,d\xi. \tag{2.7}$$

cf. Eqs. (15.9, 10), Ref. 10, so that the (n.b.) input to the following discriminator (cf. Fig. 1) is

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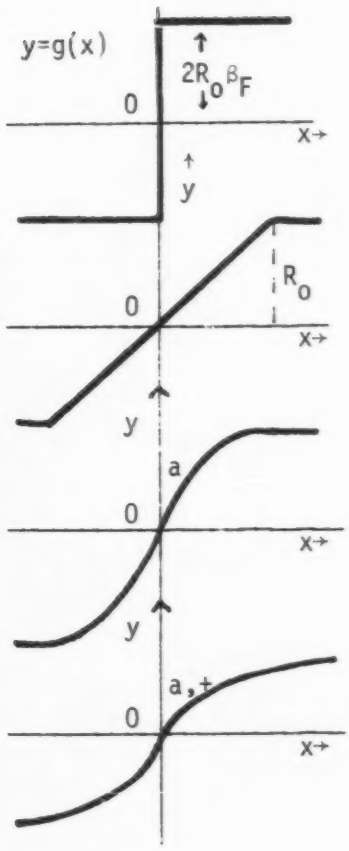
$$V_{in\text{-}d}(t) = \left|\frac{i}{\pi}\int_C f_L(i\xi)J_1(\xi E)d\xi\ \cos[\omega_o t-\theta(t)]\right| \quad . \tag{2.8}$$

Various specific limiter characteristics are sketched in Fig. 2 below, along with their corresponding transforms, f_L (including Eqs. (2.9a),(2.10)).



$$y=2R_o\beta_F \begin{cases} 1, x>0 \\ -1, x<0 \end{cases} \qquad f_L(i\xi) = \frac{2\beta_F R_o}{i\xi}$$

"Superclipper" ≡ ideal limiter (2.9a)

$$y=2\beta_F|x|, 0<|x|<R_o \qquad f_L(i\xi)=2\beta_F\left(\frac{1-e^{-i\xi R_o}}{(i\xi)^2}\right)$$
$$=2\beta_F R_o,\ |x|>R_o$$

"rectangular clipper" (2.9b)

$$y=2\beta_F\Theta(ax)$$

$$(\Theta(z)\equiv(2/\sqrt{\pi})\int_0^z e^{-t^2}dt) \qquad f_L(i\xi)=2\beta_F\{\sqrt{\frac{\pi}{4a^2}}[e^{-\xi^2/4a^2} - \frac{i\xi}{a}\,{}_1F_1(1;3/2;-\xi^2/4a^2)]\}$$

"error-function" clipper (2.9c)

$$y=2\beta_F(1-e^{-a/x}), x\geq 0 \qquad f_L(i\xi)=2\beta_F\,\frac{a}{i\xi(i\xi+a)}\ ,$$
$$=-2\beta_F(1-e^{ax}),\ x\leq 0$$

(2.9d)

"linear exponential clipper"

where

$$f_L(i\xi)=\int_0^\infty e^{-i\xi x}g(x)dx\ ,$$

and $(\xi)<0$. (2.10)

Figure 2. Various symmetrical limiters (or antisymmetrical rectifiers), cf. Sec. 13.4-2, (2) [10]. Here $f_L(i\xi)$ is the transform of the half-wave rectifier response, g(x), cf. (2.10).

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Important limiting cases are:

"Superclipping": [$a\to\infty$; $R_o\to 0$, etc.]: $B_1(E) = \left(\begin{array}{c} \frac{4\beta_F R_o}{\pi} \\ \frac{2\beta_F\sqrt{\pi}}{a} \\ \frac{4\beta_F}{\pi} \end{array} \right) \cdot 1. \quad \therefore B_1(E) \sim$ constant (2.11)

No limiting: $R_o\to\infty$: (2.9b): $B_1(E) = \beta_F R$. (2.12)

2.3. The Non-Ideal Discriminator

Here we need to extend the earlier results [of Sec. 15.1-3, Ref. 10], to construct a working model of the non-ideal discriminator's dynamic characteristic. This latter is sketched below in Fig. 3, typically. A detailed characteristic is beyond the scope of our analysis, nor is it necessary. We can follow the argument of Sec. 15.1-4, Ref. 10, and in addition use an ad hoc analytical representation, to obtain for the IF output of the discriminator

$$V_{o-d|IF} = \kappa |V_{i-d}| \left. \frac{\dot{\Phi}}{1+b^2|\dot{\Phi}|^{2\nu}} \right|_{IF}, \quad (\nu > \tfrac{1}{2},\ \kappa > 0,\ b \text{ real}), \tag{2.13}$$

where $V_{i-d} = B_1(E)$, cf. (2.7), (2.8), and $\dot{\Phi} = \omega_o - \dot{\theta}$. Only the low-frequency (spectral) "zone" is of concern to us, so we replace $\dot{\Phi}_{IF}$ by $\dot{\Phi}_{LF} = -\dot{\theta}$ in (2.13), to obtain finally the new result

$$E_o(t) = \frac{\kappa\dot{\theta}B_1(E)}{1+b^2|\dot{\theta}|^{2\nu}}, \quad \nu > \tfrac{1}{2}, \tag{2.14}$$

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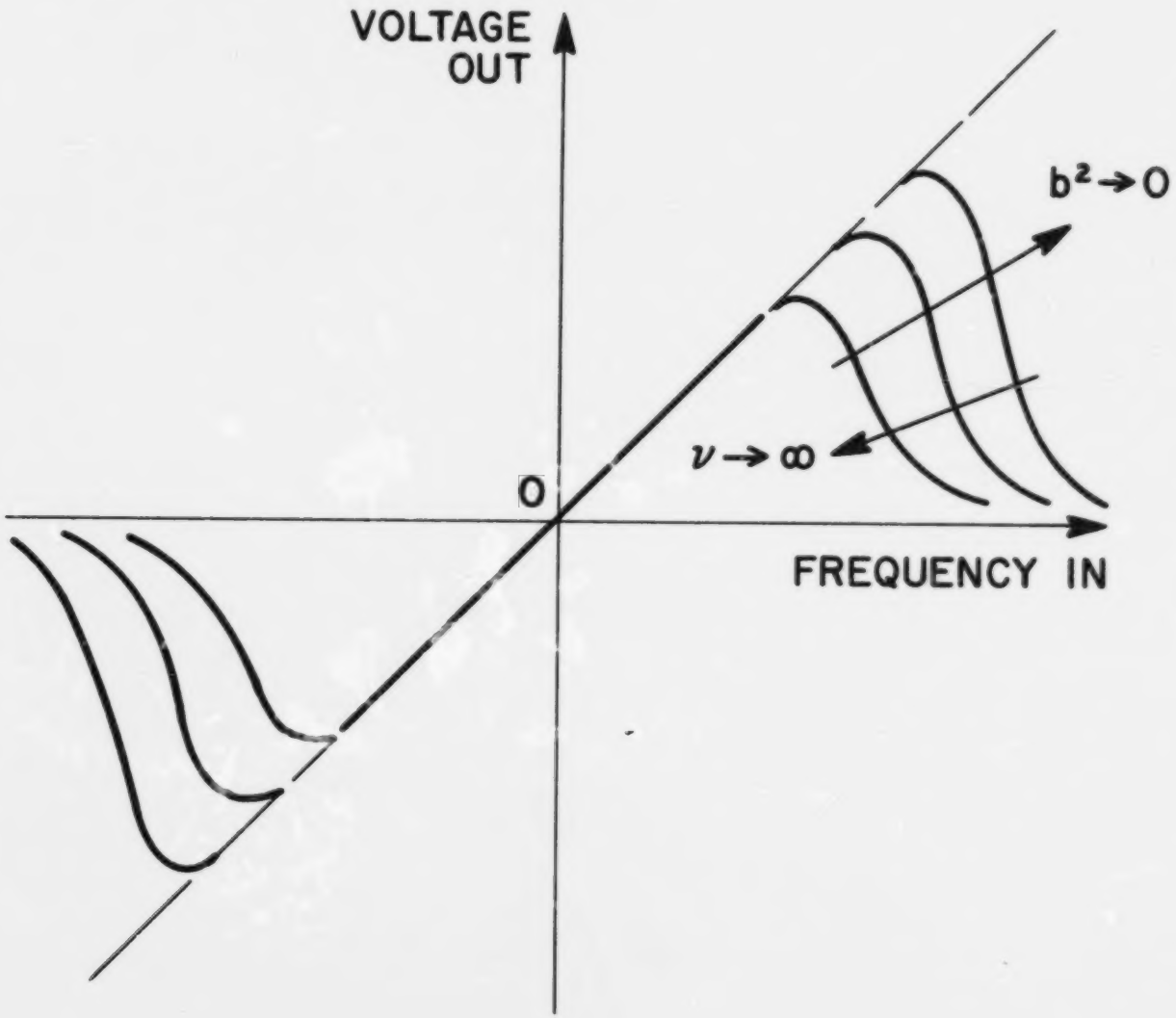

Figure 3. Dynamic characteristics of a typical, non-ideal discriminator.

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where we have now absorbed the (-) into κ. [We assume for (2.14) that the "companding" or "audio" filter, which eliminates the IF and higher spectral zones in the general discriminator output, does not distort this output, $E_o(t)$ (by frequency selection). When there is "companding", i.e., (low-pass) filtering of the discriminator output, we have, instead of (2.14),

$$E_c(t) = \int_{-\infty}^{\infty} E_o(t) h_L(t-\tau)d\tau = \int_{-\infty}^{\infty} \frac{\kappa B_1(E(t))\dot{\theta}(t)}{1+b^2|\dot{\theta}(t)|^{2\nu}} h_L(t-\tau)d\tau. \qquad (2.15)$$

Our principal task, so far, remains to obtain $E_o(t)$ explicitly, in terms of the original input $V_{in}(t)$, via (2.2)-(2.6) above, cf. Sections 3,4ff. Observe that when the corresponding filter is "wide", e.g., $h_L=\delta(\tau-0)$, $E_c(t) = E_o(t)$, as expected: the low-frequency output E_o is passed without distortion.

We note re (2.14) that $V_{o-d} = |V_{i-d}|g(\dot{\phi})$ rather than $V_{o-d} = g(|V_{i-d}|\dot{\phi})$, since when $|\dot{\phi}|$ is bounded and $|V_{i-d}|\to\infty$, $V_{o-d}=g(|V_{i-d}|\dot{\phi})\to 0$, which is <u>not</u> physically the case: increasing $|V_{i-d}|$ actually increases V_{o-d} (until the element saturates, in reality). Finally, with the three discriminator parameters (κ,ν,b^2) it should be possible, cf. Fig. 3, to fit any reasonable actual characteristic: κ for overall scale; ν for the "sharpness" of the fall-off beyond the maximum; and b^2 for the relative height-to-width of the response. Thus, Equation (2.14) is the key, non-linear function of the instantaneous <u>input</u>, whose properties we wish to evaluate.

3. MEASURES OF THE EM RECEIVER OUTPUT

We distinguish two basic measures of FM receiver output: (I), instantaneous quantities, and their associated <u>time averages</u>, $\langle\ \rangle_t$; and (II), statistical averages, $\langle\ \rangle$. Thus in the general case, based on (2.14), we have for the m^{th} (time-) moment of the instantaneous output

$$\langle E_o(t)^m\rangle_t = \lim_{T\to\infty} \int_0^T \frac{1}{T} E_o(t)^m dt = \frac{1}{T_o}\int_0^{T_o} E_o(t)^m dt, \qquad (3.1a)$$

for periodic signals, and

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$$\therefore \langle E_o(t)^m \rangle_t = \frac{\kappa}{T_o} \int_0^{T_o} \left[\frac{B_1(E)\dot{\theta}}{1+b^2|\dot{\theta}|^{2\nu}} \right]^m dt, \qquad (3.1b)$$

explicitly for such periodic signals. (Here T_o is the period of the desired signal, not necessarily that of any accompanying interference.)

Similarly, for the statistical approach, we have directly for the $m^{\underline{th}}$ moment of E_o:

$$\langle E_o(t)^m \rangle = \int_0^\infty B_1(E)^m dE \int_{-\infty}^\infty \dot{\theta}^m (1+b^2|\dot{\theta}|^{2\nu})^{-m} w_1(E,\dot{\theta})_{S+I} d\dot{\theta}. \qquad (3.2)$$

The central problem here is the explicit evaluation of $w_1(E,\dot{\theta})_{S+I}$, i.e., the joint pdf of the (input) envelope and phase derivative (≡ instantaneous frequency), into the non-linear portions of the receiver, cf. Fig. (2.1) above.

The moments of chief interest here, in addition to $\langle E_o \rangle$, $\langle \dot{E}_o^2 \rangle$, are

$$\langle E_o(t_1)E_o(t_2) \rangle = M_{E_o}(|t_2-t_1|), \text{ and } R_{E_o}(|t_2-t_1|) \equiv \langle E_o(t_1)E_o(t_2) \rangle_t, \qquad (3.3)$$

viz. the (auto-) covariance of E, and the auto-correlation function of E. From these we can obtain the (intensity) spectrum of the output, $E_o(t)$, as well. [This becomes important in the study of companding, cf. (2.15), and for the cases of broad-band FM, cf. Secs. 5.4, 15.5, Ref. 10.

In the present study, we shall consider only the instantaneous approach, and associated time averages. A quantity of central interest is the departure from the ideal output, $\dot{\theta}_o(t)$, which is what the receiver would provide (at the low-frequency, "observer" region), were there (i), no interfering signals; (ii), no departures from superclipping in the limiter; (iii), no departures from ideal discrimination, i.e. $b^2=0$. Thus, the ideal output is

$$\dot{E}_o(t) = \kappa\beta_F, \dot{\theta}_o(t) = \kappa\beta_F' \dot{\theta}(t)\Big|_{\text{no interference}},$$

$$\therefore \dot{\theta}(t)_{\text{no int.}} = \dot{\theta}_o(t). \qquad (3.4)$$

For example, in the case of interference, superclipping, and ideal discrimination, we have from (2.14) (and the results of Sections (4,5) ff):

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$$E_o(t) = \frac{\kappa\beta_F R}{\pi}\left|\frac{\dot{\theta}_o}{1+b_I(t)} + \frac{+\omega_I(t)}{1+b_I(t)}\right| \quad , \quad \text{cf. (4.14), (4.15), ff., (3.5)}$$

where $b_I(t)$ and $\omega_I(t)$ are respectively the modifications in the envelope and "frequency" portions of $\dot{\theta}$, produced by the interference accompanying the desired signal. We shall see specifically just what the structures of $b_I(t)$, $\omega_I(t)$ are, in Section 4 following.

4. INSTANTANEOUS FREQUENCY AND ENVELOPE INPUTS TO THE NONLINEAR STAGES OF THE RECEIVER

We next extend the signal "input" model to the limit-discriminator of Section 2, cf. (2.4)-(2.6), to include an additive set of arbitrary interference waves. Let

$$A_j(t)e^{i\Delta\omega_j t - i\Psi_j(t)} = e^{i\Delta\omega_j t}\int_{-\infty}^{\infty}\{B_j(t-\tau)e^{-i\phi_j(t-\tau)}\}\{h_o e^{-i\gamma_o - i\Delta\omega_j\tau}\}_\tau d\tau \tag{4.1}$$

where j designates the j^{th} input signal, both to the ARI stages of the receiver, e.g.,

$$B_j(t)e^{i\Delta\omega_j t - i\phi_j(t)} = \alpha_{in}(t) - \beta_{in}(t) \quad , \quad j = 1,\ldots,M \quad , \tag{4.2}$$

for the (undistorted) inputs, while (4.1) gives the corresponding j^{th} output of the ARI-stages (≡ <u>input</u> to the limiter-discriminator (L-D), etc.). Accordingly, the totality of input signals to the receiver is

$$\mathrm{Re}\{e^{i\omega_c t}\sum_{j=0}^{M} B_j(t)e^{i\Delta\omega_j t - i\Phi_j(t)}\} = V_{in}(t),$$

while

$$\mathrm{Re}\{e^{i\omega_o t}\sum_{j=0}^{M} A_j(t)e^{i\Delta\omega_j t - i\Psi_j(t)}\} = V_o(t) \tag{4.3b}$$

is the desired input to the L-D portions of the receiver. In fact, from (2.5a) with (4.3b), we see at once that

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$$\alpha_o(t)-i\beta_o(t) = \sum_{j=0}^{M} A_j(t)e^{i\Delta\omega_j t - i\Psi_j(t)} \quad ,$$

$$(\Delta\omega_j = \omega_{c_j} - \omega_o = \omega_{D_j}), \qquad (4.4)$$

so that

$$E(t) = \sqrt{\alpha_o^2 + \beta_o^2} \quad ; \quad \theta(t) = \tan^{-1}(\beta_o/\alpha_o) \; , \qquad (4.4a)$$

cf. (2.5a).

We introduce the following conventions:

(i). j=0: the desired signal; $\Delta\omega_j = 0$ (usually the receiver is in tune with the desired signal).

(ii). $j\neq 0$ ($j\geq 1$), (or $j<0$),; $\Delta\omega_j \neq 0$ (or $\Delta\omega_j=0$); usually the interference is "off-tune"; but, of course we can treat the special "on-tune" cases: $\Delta\omega_j=0$; $j\geq 1$, etc.

(iii). $j\neq 0$; $j \rightarrow -j$; we use <u>negative indexes</u> to indicate interfering signals, below the carrier (central) frequency f_o, cf. Fig. 4. Thus, suppose we have M pairs of interfering signals; then

$$\sum_{j=0}^{M} \rightarrow \sum_{0}^{2M}$$

in (4.4). Or if there is just one interference, below f_o, say j'; we write it $A_{-j'}e^{i\Delta\omega(-j')t-i\Psi(-j')}$, etc.

In this way it is a very simple matter to designate the various possible spectral locations, as well as wave-form character of the various input signals. Note that because of the non-ideality of the ARI (linear) filter stage, i.e., because $h_o e^{-i\gamma_o} \neq \delta(\tau-0)$ - i.e., does not pass all frequencies of the input equally, there is <u>distortion</u>, produced by this filter. Hence, even if the B's were constant amplitudes, and therefore only angle-modulations were present ($\sim\Phi_j$'s), there would be some amplitude modulations in the filter output, i.e., $A_j = A_j(t)$. The specific nature of this effect has been obtained

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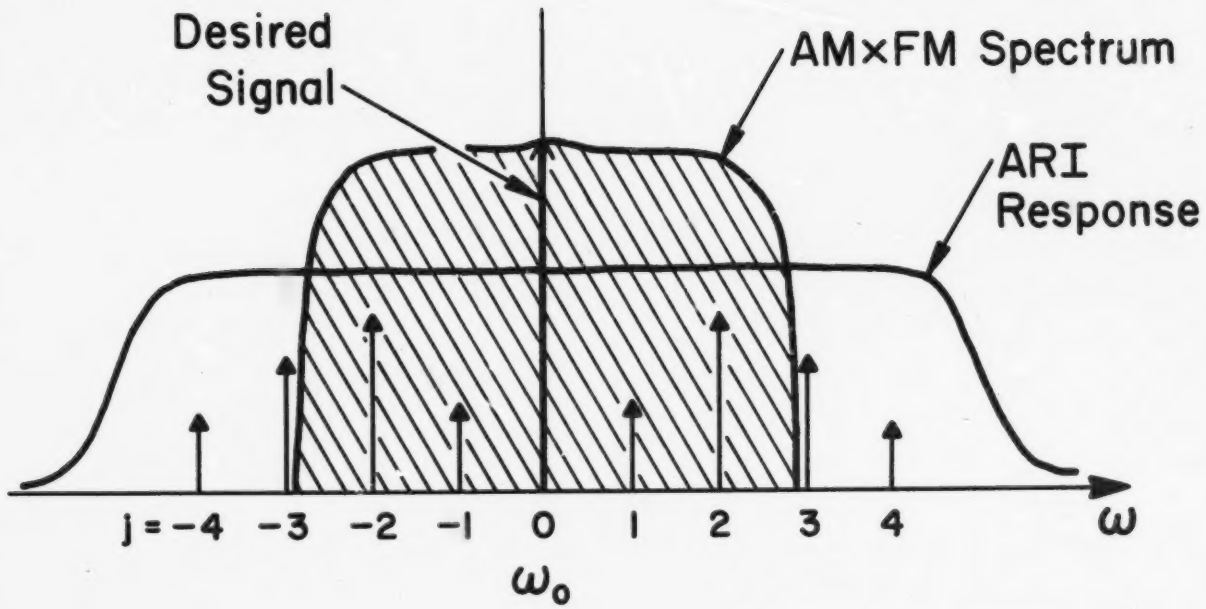

Figure 4. Sketch of intantaneous spectrum of desired signal, and interfering carriers (modulations present, but not shown).

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by Bedrosian and Rice[11], which enables us to relate (B_j,Φ_j) to the corresponding (A_j,Ψ_j) and vice versa. (We treat this effect explicitly in Ref. [6].)

To use (2.14), (or (2.15), we need explicit expressions for E and $\dot{\theta}$, which are obtained from (4.4), (4.4a), viz:

$$E(t)^2 = \alpha_o^2+\beta_o^2 \quad ; \text{ with } \alpha_o = \sum_j A_j(t)\cos(\Delta\omega_j t-\Psi_j);$$

$$\beta_o = \sum_j A_j(t)\sin(\Delta\omega_j t-\Psi_j), \tag{4.5}$$

and

$$\dot{\theta} = \frac{d}{dt}\tan^{-1}(\beta_o/\alpha_o)= \frac{\alpha_o\dot{\beta}_o-\dot{\alpha}_o\beta_o}{\alpha_o^2+\beta_o^2} \quad . \tag{4.6}$$

The results are

$$E^2(t)= \sum_{jk} A_jA_k\cos\psi_{jk} \quad ; \begin{cases} \psi_{jk}(t)\equiv(\Delta\omega_j-\Delta\omega_k)t-\Psi_j+\Psi_k, \text{ with} & (4.7) \\ \Psi_j(t) = \hat{\Psi}_j+\phi_j; & (4.7a) \end{cases}$$

$$\dot{\theta}(t) = \dot{\mathcal{P}}_{\dot{\theta}}/E^2 \text{ , with} \tag{4.8a}$$

$$\left.\begin{aligned} \dot{\mathcal{P}}_{\dot{\theta}} &\equiv \alpha_o\dot{\beta}_o-\dot{\alpha}_o\beta_o \quad (\equiv \text{"instantaneous frequency factor"}) \\ &= \sum_{jk} A_j(t)A_k(t)\Omega_{jk}(t)\cos(\psi_{jk}-\eta_{jk}) \quad , \end{aligned}\right\} \begin{matrix} (4.8b) \\ (4.8c) \end{matrix}$$

where, specifically

$$\Omega_{jk}(t) = [(\Delta\omega_j-\dot{\Psi}_j)^2+\{\tfrac{1}{2}\frac{d}{dt}(\log A_k-\log A_j)\}^2]^{1/2} \tag{4.9a}$$

$$\eta_{jk}(t) = \tan^{-1}\left\{\frac{-(\dot{\log} A_k-\dot{\log} A_j)/2}{\Delta\omega_j-\dot{\Psi}_j}\right\} \quad ; \quad \dot{\log} A_k \equiv \frac{d}{dt}\log A_k, \text{ etc.} \tag{4.9b}$$

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In matrix notation we can write

$$\underset{\sim}{b}_I = [(b_I)_{jk}] \equiv [\cos \psi_{jk}] \; ; \; \underset{\sim}{A} = [A_j] \quad (\geq 0); \tag{4.10a}$$

$$\underset{\sim}{\omega}_I = [(\omega_I)_{jk}] \equiv [\Omega_{jk} \cos (\psi_{jk} - \eta_{jk})] \; , \tag{4.10b}$$

so that (4.7), (4.8) become, more compactly

$$E^2(t) = \tilde{\underset{\sim}{A}} \, \underset{\sim}{b}_I \underset{\sim}{A} \; ; \; \dot{\mathcal{P}}_\theta^2 = \tilde{\underset{\sim}{A}} \, \underset{\sim}{\omega}_I \underset{\sim}{A} \; ; \therefore \; \dot{\theta}(t) = \dot{\mathcal{P}}_\theta / E^2 = \frac{\tilde{\underset{\sim}{A}} \underset{\sim}{\omega}_I \underset{\sim}{A}}{\tilde{\underset{\sim}{A}} \underset{\sim}{b}_I \underset{\sim}{A}} \tag{4.11}$$

We note, also, that while $E^2 = \tilde{\underset{\sim}{A}} \underset{\sim}{b}_I \underset{\sim}{A} \geq 0$, i.e., is positive semi-definite ($E=0$ for some t), $\dot{\mathcal{P}}_\theta$ can be positive, negative, or zero, again for some t. In addition, we remark from (4.7a) that the interference elements ($j \neq 0$) have <u>different phases relative to the desired input signal</u> ($j=0$), as indicated by ϕ_j ($\neq 0$), generally. This feature must be observed when we come to calculate time averages, or instantaneous values, of $E_o(t)$.

To analyze further the character of the interference as it appears in E, $\dot{\theta}$, from an analytical viewpoint, it may be convenient to separate the desired signal effects from those produced by the interference. Accordingly, (keeping the proper dimensions in our notation), from (4.7), we let

(4.12a)
$$\begin{cases} S_E^2 \equiv A_o(t)^2: \text{ distorted, desired signal contribution (to the envelope); } (\psi_{oo}=0; \eta_{oo}=0). \\ I_E^2 \equiv \sum_{jk}'' A_j A_k \cos\psi_{jk}: \text{ interference effects (in the envelope) produced solely by the input other than the desired signal, and independent of it; } (j=k=0 \text{ excluded}). \\ (SI)_E \equiv 2A_o \sum_k' A_k \cos\psi_{ok}: \text{ cross-interference, between desired signal and the other input interference waves } (j=0); \; k \geq 1, \text{ produced in the envelope.} \end{cases}$$

Similarly, for $\dot{\mathcal{P}}_\theta$ we can write, cf. (4.8c)

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$$(4.12b)\begin{cases} \dfrac{dS^2_{\dot\theta}}{dt} = -A_o(t)^2\dot\psi_o(t): \text{ distorted desired signal contribution to the instantaneous frequency factor } (\dot\rho_{\dot\theta}) \text{ of the instantaneous frequency, (4.8b), } (\eta_{oo}=0). \\ \dfrac{dI^2_{\dot\theta}}{dt} = \sum_{jk}{}'' A_jA_k\Omega_{jk}\cos(\psi_{jk}-\eta_{jk}): \text{ interference effects (in } \dot\rho_{\dot\theta}) \text{ generated solely by the incoming interference, and independent of the desired signal } (j=0) \text{ with } (j,k)\neq(j=k=0). \\ \dfrac{d(SI)_{\dot\theta}}{dt} = 2A_o \sum_{k=1}{}' A_k\Omega_{ok}\cos(\psi_{ok}-\eta_{ok}): \text{ cross-interference, between signal and interference, in } \dot\rho_{\dot\theta}. \end{cases}$$

Examination of (4.12a,b) accordingly gives further insight into the nature of the instantaneous output (angular) frequency $\dot\theta$. Combining (4.12a,b) in (4.11) and dividing by A_o^2 in numerator and denominator permits us to write

$$\boxed{\dot\theta = \frac{-\dot\psi_o + \sum_{j,k}{}^{(1,1)} a_ja_k\Omega_{jk}\cos(\psi_{jk}-\eta_{jk})}{[1+\sum_{jk}{}^{(1,1)} a_ja_k\cos\psi_{jk}](\equiv E^2(t)/A_o^2(t))>0}}\ , \ (\Delta\omega_o=0, \text{ etc.}), \tag{4.13}$$

where

$$a_{j,k}(t) \equiv A_{j,k}/A_o \quad (\geq 0), \text{ and } \left\{ {}^{(1,1)} \equiv \text{the term } j=k=0, \text{ only, is omitted in } \sum_{jk}{}^{(1,1)} \right\} \tag{4.13a}$$

The result (4.13) can also be written

$$\dot\theta = \frac{\dot\theta_o}{1+b_I} + \frac{\omega_I}{1+b_I}\ , \text{ cf. (3.5)}; \ \dot\theta_o = -\dot\phi_o, \text{ cf. (4.2)}\ , \tag{4.14}$$

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where now at once from (4.13) we see that

$$(1+b_I)(\equiv E^2/A_o^2) \equiv 1+ \sum_{jk}^{(1,1)} a_j a_k \cos\psi_{jk} \; ; \quad \omega_I \equiv (\dot{\Phi}_o - \dot{\Psi}_o) + \sum_{jk}^{(1,1)} a_j a_k \Omega_{jk} \cos(\psi_{jk} - \eta_{jk}) \tag{4.15}$$

When there is no distortion, i.e. $h_o = \delta(\tau-0)$, or equivalently, when the ARI-linear filter is wide enough to pass the input without modification, we readily see that (4.13) reduces directly to

$$\dot{\theta}\Big|_{\text{no dist.}} = \frac{-\dot{\Phi}_o + \sum_{jk}^{(1,1)} b_j b_k \Omega_{jk} \cos(\psi_{jk} - \eta_{jk})}{(1+ \sum_{jk}^{(1,1)} b_j b_k \cos\psi_{jk}) \; (\geq 0)} \; , \tag{4.16}$$

where

$$-1 \leq b_j(t) (\equiv B_j(t)/B_o(t)) \leq 1, \tag{4.16a}$$

and where

$$E(t)^2\Big|_{\text{no dist.}} = B_o(t)^2 [1+ \sum_{jk}^{(1,1)} b_j(t) b_k(t) \cos\psi_{jk}(t)]. \tag{4.16b}$$

When there is no original amplitude modulation, e.g. $B_o(t)=B_j(t)= B_o, B_j$, etc., = constants (≥ 0), (4.16)-(4.16b) reduce to the simpler forms

$$\dot{\theta}(t)\Big|_{\substack{\text{n.d.} \\ \text{n.AM}}} = \frac{-\dot{\Phi}_o + \sum_{jk}^{(1,1)} b_j b_k (\Delta\omega_j - \dot{\Phi}_j) \cos\psi_{jk}}{[1 + \sum_{jk}^{(1,1)} b_j b_k \cos\psi_{jk}](\equiv E^2(t)/B_o^2)} \; , \tag{4.17}$$

with now the b_j's (and $\Delta\omega_j$) independent of time. Equations (4.16), (4.17) are the fundamental expressions for $\dot{\theta}$ and E^2 which we shall need to evaluate in this initial study.

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The quantities (b_I,ω_I), cf. (4.14), reduce for (4.16), (4.17), to

$$b_I(t)\Big|_{\text{no dist.}} = 1+\sum_{jk}^{(1,1)} b_j(t)b_k(t)\cos\psi_{jk} \quad ;$$

$$\omega_I(t)\Big|_{\text{no dist.}} = \sum_{jk}^{(1,1)} b_j(t)b_k(t)\Omega_{jk}\cos(\psi_{jk}-\eta_{jk}), \tag{4.18a}$$

and, for (4.17),

$$b_I(t)\Big|_{\substack{\text{n.d.}\\ \text{n.AM}}} = 1+\sum_{jk}^{(1,1)} b_j b_k \cos\psi_{jk}$$

$$\omega_I(t) = \sum_{jk}^{(1,1)} b_j b_k(\Delta\omega_j-\dot{\Phi}_j)\cos\psi_{jk}\ , \tag{4.18b}$$

where now the b_j are constants.

Still other simplifications, as the case may warrant, can be made. For example, let us suppose that in addition to (i), no distortion, and (ii), no amplitude modulation, we require all angle modulations, including that of the desired signal, to be the same, e.g.

$$\Psi_j = \Psi_o \to \Phi_o, \text{ here; all } (j,k). \tag{4.19a}$$

Then, we have

$$\therefore\ \psi_{jk} = (\Delta\omega_j-\Delta\omega_k)t, \text{ cf. (4.7)}\ ;\ \eta_{jk} = 0, \text{ as before,} \tag{4.19b}$$

and (4.17b) reduces further to

$$\dot{\theta}\Big|_{\substack{\text{no dist}\\ \text{no amp. mod}\\ \text{equal mod}}} = \frac{-\dot{\Phi}_o+\sum_{jk}^{(1,1)} b_j b_k(\Delta\omega_j-\dot{\Phi}_o)\cos(\Delta\omega_j-\Delta\omega_k)t}{[1+\sum_{jk}^{(1,1)} b_j b_k\cos(\Delta\omega_j-\Delta\omega_k)t]\,(\cong E^2/B_o^2)^2}\ , \tag{4.20}$$

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where, as before, cf. (4.17b), $b_j, b_k (\geq 0)$ are (real) constants. Since now ($b_k^* = b_k$, real) and

$$b_I \equiv \sum_{jk}^{1,1} b_j b_k \cos(\Delta\omega_j - \Delta\omega_k)t = \mathrm{Re} \sum_{jk}^{(1,1)} b_j b_k^* e^{i\Delta\omega_j t - i\Delta\omega_k t}$$

$$= \mathrm{Re}\left| \sum_0^M b_j e^{i\Delta\omega_j t} \right|^2 - b_o^2 \; ; \; b_o = 1,$$

$$\therefore \; b_I = \left| \sum_0^M b_j e^{i\Delta\omega_j t} \right|^2 - 1 \; , \tag{4.21}$$

we may interpret the series in (4.21) as simply the scalar length of the vector resultant of the (frequency) displacement vectors of the various "off-tune" interfering carriers ($\omega_o + \Delta\omega_j$) about ω_o. Note that (4.20) can be rewritten

$$\dot{\theta}\Big|_{\substack{\neq \text{dist.} \\ \neq \text{amp.mod} \\ \text{equal mod}}} = -\dot{\Phi}_o + \frac{\sum_{jk}^{(1,1)} b_j b_k \Delta\omega_j \cos(\Delta\omega_j - \Delta\omega_k)t}{[1 + \sum_{jk}^{(1,1)} b_j b_k \cos(\Delta\omega_j - \Delta\omega_k)t](\equiv E^2/B_o^2)} \quad \left(= \dot{\theta}_o + \frac{\omega_I}{1+b_I}\right), \tag{4.22}$$

cf. (4.14). Other special situations are readily examined along similar lines, cf. Sec. 6.6.

It is convenient to renormalize (4.16), (4.17) in terms of the total instantaneous signal intensity (TISI) of the interference, e.g.

$$b_M^2 B_o^2 \; (= b_M^2(t) B_o^2(t)) \equiv \sum_{j=1}^M B_j(t)^2 \quad , \tag{4.23a}$$

$$\therefore \; b_M(t)^2 = \sum_{j=1}^M B_j(t)^2 / B_o^2(t) \; . \tag{4.22b}$$

We have for (4.16)

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$$\dot{\theta}(t)\Big|_{n.d.} = \frac{-\dot{\Phi}_o + b_M^2 \sum_{jk}^{(1,1)} \hat{b}_j \hat{b}_k \Omega_{jk} \cos(\psi_{jk} - \eta_{jk})}{[1 + b_M^2 \sum_{jk}^{(1,1)} \hat{b}_j \hat{b}_k \cos \psi_{jk}]}, \quad (-1 \leq \hat{b}_{j,k} \leq 1), \tag{4.24}$$

with

$$\hat{b}_j(t) \equiv b_j(t)/b_M = \frac{B_j/B_o}{[\sum_{j=1}^{M} B_j^2(t)/B_o^2]^{1/2}} = B_j/(\sum_{j=1} B_j^2)^{1/2}, \tag{4.24a}$$

where, of course, the $(b_j, \hat{b}_j; b_M, B_j)$ are generally functions of time. In the important subcase of no amplitude modulation as well as no front-end distortion, (4.17) reduces to

$$\dot{\theta}(t)\Big|_{\substack{n.d. \\ n.AM}} = \frac{-\dot{\Phi}_o + b_M^2 \sum_{jk}^{(1,1)} \hat{b}_j \hat{b}_k (\Delta\omega_j - \dot{\Phi}_j) \cos \psi_{jk}}{[1 + b_M^2 \sum_{jk}^{(1,1)} \hat{b}_j \hat{b}_k \cos \psi_{jk}]}, \quad (-1 \leq \hat{b}_{j,k} \leq 1), \tag{4.25}$$

similarly, where now the $(b_M, b_j, \hat{b}_j, B_j)$ <u>are constants</u>, however.

Finally, corresponding normalizations of the b_j's in the other special cases (4.20)-(4.22) follow immediately from (4.23)-(4.25) suitably specialized to these cases.

5. VARIOUS INSTANTANEOUS RECEIVER OUTPUT MODELS

Depending on the constraints imposed on our FM receiver model, as well as on the detailed structure of the ipput, we can obtain a hierarchy of output models, each of interest and importance in its own right, ranging from the simplest to the (almost) most general, (2.14).

Thus, from (2.14) we may specialize successively to:

Class 0. <u>The Ideal Receiver</u> $E_o = \alpha\hat{\theta} = \alpha\dot{\theta}_o$; (no interference); (5.1)

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Class I. The Ideal Receiver
[superclipper-ideal discrim.] $E_0 = \alpha\dot{\theta}$, with interference; (5.2)

Class II. No Limiting-Ideal Discrim. $E_0 = \alpha E\dot{\theta}$ ← (all with interference); (5.3)

Class III. Superclipping-Non Ideal Discrim. $$E_0 = \frac{\alpha\dot{\theta}}{1+b^2|\dot{\theta}|^{2\nu}} ;$$ (5.4)

Class IV. No Limiting-Non Ideal Discrim. $$E_0 = \frac{\alpha E\dot{\theta}}{1+b^2|\dot{\theta}|^{2\nu}} ;$$ (5.5)

Class V. Arbitrary Limiting-Ideal Discrim. $E_0 = \alpha B_1(E)\dot{\theta}$; (5.6)

Class VI. Arbitrary Limiting-Non Ideal Discrim. $$E_0 = \frac{\alpha B_1(E)\dot{\theta}}{1+b^2|\dot{\theta}|^{2\nu}} ,$$ (5.7)

For our purposes Class VI, (5.7), is the most general FM receiver output we shall ultimately consider quantitatively.

Since E and $\mathcal{P}_{\dot{\theta}}$ are explicitly given by (4.7), (4.8a), etc., with $\dot{\theta}$ generally given by (4.13), we have all the ingredients to

(i). evaluate the instantaneous output $E_0(t)$;
(ii). its various m^{th}-moments;
(iii). take time averages, $\langle E_0(t)^m \rangle$.

If we confine our attention to general periodic modulations, typically chosen analytic forms, e.g. $\Psi_j = \Psi_j(t)$ given, with selected amplitudes and frequency displacements, $\Delta\omega_j$, etc., we can obtain the desired time averages from the instantaneous values in a straightforeward way by direct numerical integration of E_0, once $E_0(t)$ has been computed for $t \in T_0$, in a period, T_0, of the desired signal modulation. [Care must be taken in the vicinity of $E(t) = 0$ in $(0,T_0)$, as then $\dot{\theta} \to \pm\infty$, usually.] Thus, we have

$$\langle E_0^m \rangle_t = \frac{1}{T_0}\int_0^{T_0} E_0(t)^m_{per}\Big|_{j=0} dt \doteq \frac{1}{n}\sum_{\ell} E_0(t_\ell)^m \text{ , etc.} \qquad (5.8)$$

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The above is the procedure followed here, cf. Sec. 7.

Various avenues for simulation also suggest themselves. The most general, perhaps is to model the receiver shown in Fig. 1, with appropriately chosen limiter discriminator characteristics [cf. Figs. 2 and 3 and use (4.4) as a guide to the input signal and interference structure, assigning ranges of values to the indicated parameters, as desired. Or we can simulate various subelements of the general receiver output, viz. E, $\dot{\theta}$, according to (4.13), and then apply these to the indicated computational procedures. Accordingly, we shall examine several specific interference "scenarios" below, in Sections 6 and 7.

6. VARIOUS INTERFERENCE SCENARIOS

In order to use the general results of Section 4, we must first specify the desired FM signal and those which constitute the (FM) interference. We employ signals of two types: (1), a simple sinusoidal FM; and (2), a square-wave FM. The former is perhaps the simplest of signals, and one which readily permits identification in the output (when the interfering signals are of different frequencies), while the latter is a relatively complex (non random) waveform, with many frequencies, but with a simple analytic structure. Similarly, the interference consists of the same type of FM signal, but at somewhat different frequencies. Also, as stated in the Introduction, in all cases, we assume no front-end distortion by the receiver, so that (4.16) or (4.17), (4.24) or (4.25) apply. Also, in our numerical work (described in Section 7) we further postulate no original amplitude modulations.

Accordingly, we have

6.1 The Desired Input Signal

Specifically, we have the modulations and corresponding instantaneous phases:

A. Sinusoidal FM: $m_A(\omega_a t) = \cos \omega_a t \quad ; \quad \Phi'_{FM-A}(t) = \frac{D_F B_o}{\omega_a} \sin \omega_o t$ (6.1a)

$$\therefore \dot{\Phi}_{FM-A} = D_F B_o m_A(\omega_a t) = D_F B_o \cos \omega_a t \; ; \qquad \left.\right\} -\pi \leq \omega_a t \leq \pi. \tag{6.1b}$$

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and

$$\underline{\text{B. Square Wave FM:}}\quad m_B(\omega_a t) = +1,\ -\pi<\omega_a t<0 \qquad = -1,\ 0<\omega_a t<\pi \qquad (6.2a)$$

$$\left.\begin{aligned}\dot{\phi}_{FM\text{-}B} &= D_F B_o(\pm 1)\ ,\\ \phi'_{FM\text{-}B} &= \frac{D_F B_o}{\omega_a}\ (\pm\omega_a t+\pi/2)\end{aligned}\right\}\quad \text{in the respective intervals } (-\pi,0),(0,\pi),\ \text{cf. (6.2a).} \qquad (6.2b)$$

These modulations and instantaneous phases are illustrated in Fig. 5. It is also convenient to introduce the following parameters:

$\gamma \equiv \Delta\omega_D/\omega_a$: frequency displacement (from the desired signal carrier frequency), measured in units of the modulating frequency. (6.3a)

$\mu_F \equiv D_F B_o/\omega_a$: <u>modulation index</u> [Ref. 10, Chapter 14].

$\lambda \equiv \gamma/\mu_F = \Delta\omega_D/B_o D_F$; ($\therefore \Delta\omega_D = B_o D_F \gamma/\mu_F$). Here λ is a measure of frequency displacement (cf. 6.3a), now in terms of the maximum frequency excursionsof the signal.

6.2 The Prototypical Interfering Signal

The basic interfering signal here is chosen to be the same type as either A or B above for the desired signal, but with somewhat different periodicities, or modulating frequencies. Thus, if ω_{a1} is the fundamental (angular) frequency of the typical interfering signal, we set

$$\eta \equiv \omega_{a1}/\omega_a\ ,\quad (0<)\ \eta < 1,\ \text{or}\ \eta > 1, \qquad (6.4)$$

as a measure of frequency "mismatch". Then, analogous to (6.1), (6.2), we have

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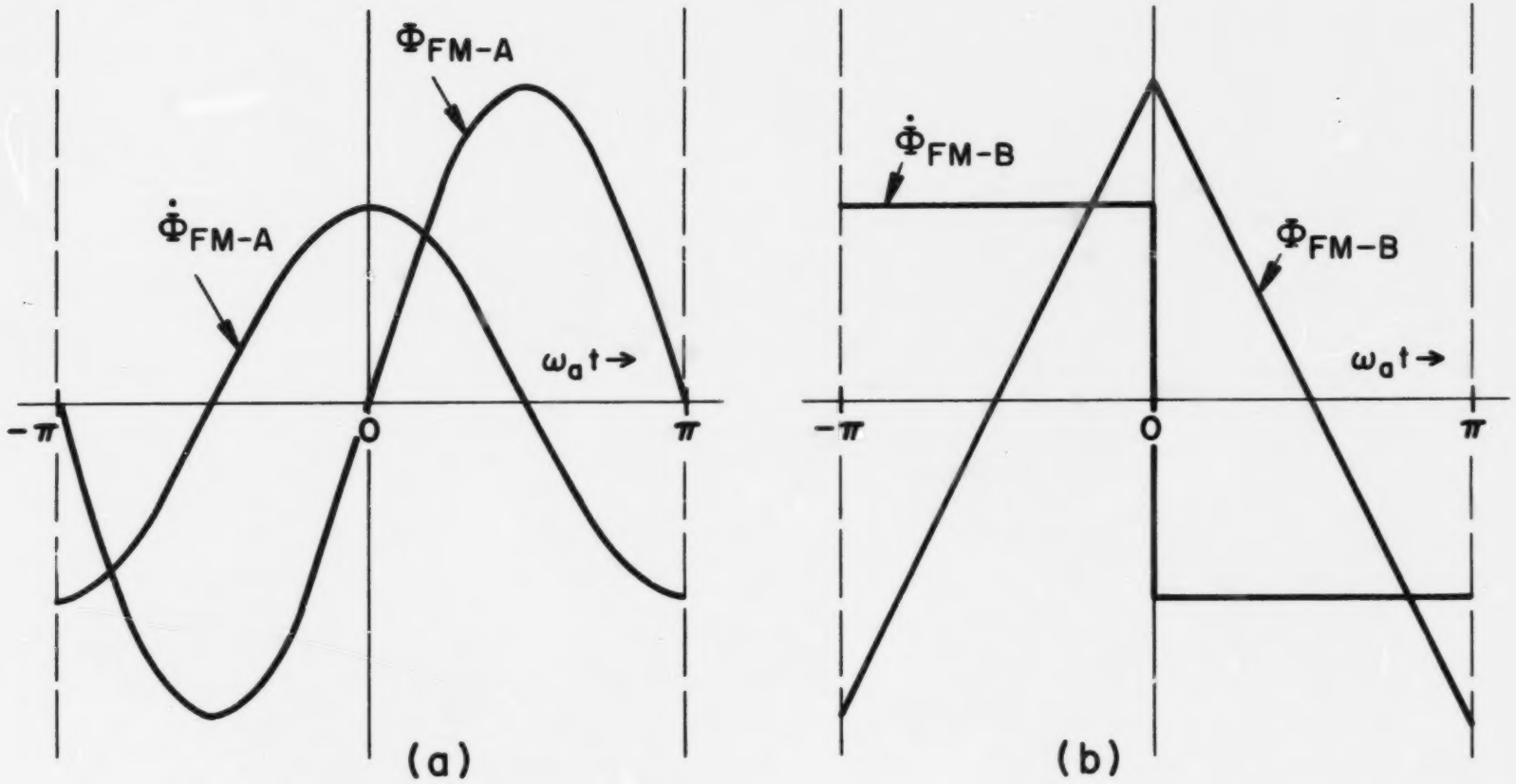

Figure 5. Desired input signals: (a) sinusoidual FM and instantaneous phase; (b) square-wave FM and instantaneous phase.

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A. Interfering Signal (A): $\dot{\Phi}_{1\text{-}A} = D_F B_o m_A(\omega_{a1} t) = D_F B_o \cos \eta\omega_a t$ (6.5a)

$$\therefore \quad \Phi'_{1\text{-}A} = \frac{D_F B_o}{\omega_{a1}} \sin\omega_{a1} t = \frac{D_F B_o}{\omega_a} \frac{\omega_a}{\omega_{a1}} \sin \omega_{a1} t = \mu_F \eta^{-1} \sin\eta\omega_a t \quad (6.5b)$$

where we have for convenience (and to reduce the number of parameters) chosen the same (max.) frequency deviations ($=D_F B_o$) as for the desired signal. (This causes no significant loss of generality in the subsequent computations, and, in any case, our relations (6.5) can readily be restored to their original generality for other computational scenarios.) Similarly, we have [cf. (6.2)]

B. Interfering Signal (B):

$$\dot{\Phi}_{1\text{-}B} = D_F B_o (\pm 1) \ , \left\{ \begin{matrix} -\pi < \omega_{a1} t < 0 \ ; \\ 0 < \omega_{a1} t < \pi \end{matrix} \right\} \text{ or } \left\{ \begin{matrix} -\pi < \eta\omega_a t < 0 \\ 0 < \eta\omega_a t < \pi \end{matrix} \right\} \quad (6.6a)$$

$$\therefore \quad \Phi'_{1\text{-}B} = \frac{D_F B_o}{\omega_a \eta} (\pm \eta\omega_a t + \pi/2), \text{ in the respective intervals.} \quad (6.6b)$$

Figures 5a, 5b apply here, provided we change the basic periodicity, i.e. period interval, according to the value of η: for $\eta > 1$ this interval is reduced; for $\eta < 1$ it is expanded. Clearly, various combinations of interference and desired signal are possible. Again, to keep the exposition manageable and without significant loss of generality, we shall select as interference the same type of modulation as the desired signal, e.g. type A with A, etc., and reserve other signal combinations to subsequent (numerical) studies.

6.3 Interference Scenario (A.1): Single Cochannel Interferer

This situation involves a single (M=1) interfering FM modulated carrier, which is "in-tune" with the desired signal. Here we set

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B_0 = amplitude of desired signal $\left.\right\}$ $\Phi_0' \equiv \Phi_0(t)$ = angle-modulation of desired signal

B_1 = amplitude of interfering signal $\left.\right\}$ $\Phi_1' = \phi_1 + \Phi_1(t)$ = angle modulation of interfering signal,

(6.7)

where we note an arbitrary phase, ϕ_1, of the interfering signal relative to the desired one, as well as an (angular) frequency displacement, $\Delta\omega_D = 0$, from the desired carrier, for this "on-tune" situation.

Setting

$$I_{M=1} = B_0^2/2 \; ; \quad S/I_{1(=M)} = (B_0^2/2)/(B_1^2/2) = B_0^2/B_1^2 \tag{6.8}$$

for an "independent", or incoherent signal-to-interference ratio between the desired and undesired signals, viz., the two signals are regarded as mutually incoherent, or uncorrelated, we find from (4.17), or (4.25), that

$$\dot{\theta}(t)\Big|_{\substack{\text{n.d.}\\ \text{n.AM}\\ \Delta\omega_D=0}} = \frac{-\{\dot{\Phi}_0(S/I_1)+\sqrt{S/I_1}\;(\dot{\Phi}_0+\dot{\phi}_1)\cos(\Phi_0'-\Phi_1-\phi_1)+\dot{\Phi}_1\}}{[(S/I_1)+2\sqrt{S/I_1}\;\cos(\Phi_0'-\Phi_1'-\phi_1)+1]\;(\equiv E^2/B_1^2)} \tag{6.8}$$

Note that as $(S/I_1)\to\infty$, $\dot{\theta}\sim-\dot{\Phi}_0$, a purely "ideal" performance, as expected, while if $(S/I_1)\neq 1$, the envelope E(t) is always positive. When $\sqrt{S/I_1} = 1$, E can become zero, so that $\dot{\theta}\to\infty$, representing a sudden jump (or "click") in phase. The signal-interference scenario is sketched in Fig. 6.

6.4 Interference Scenario (B.1): Adjacent Channel Interference by Single Source

Here we have the situation sketched in Fig. 6, where the interfering source is displaced from the desired signal (at the carrier frequency) by an amount $\Delta\omega_D$. Equation (4.17) becomes, for the condition $\Delta\omega_1 = \Delta\omega_D$, with (6.7),

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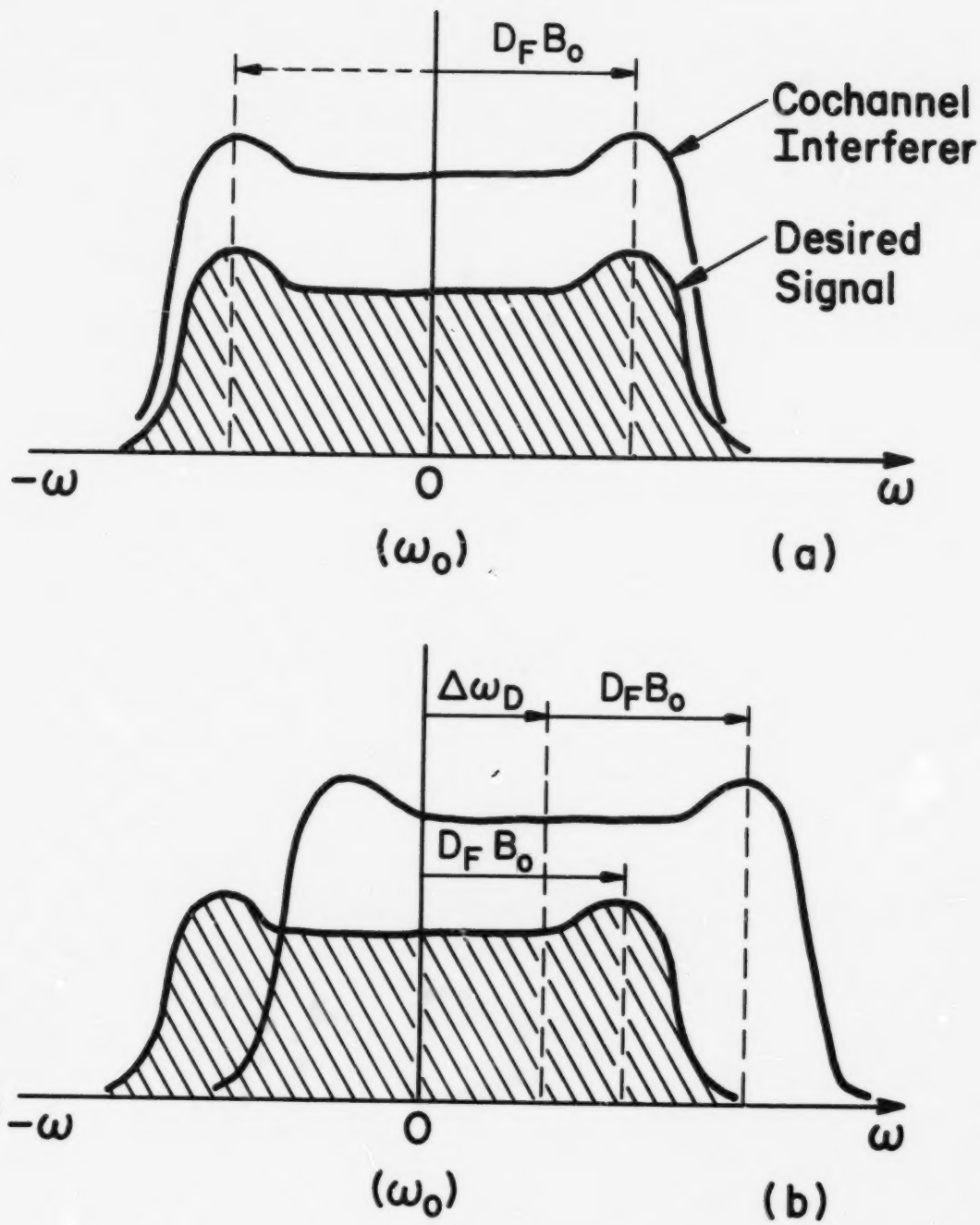

Figure 6. "Spectra" of a single cochannel and adjacent channel interferer, with "on-tuned" desired signal.

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$$\dot{\theta}(t)\Big|_{\substack{\text{n.d.}\\ \text{n.AM.}}} = \frac{-\dot{\phi}_0(S/I_1)+\sqrt{S/I_1}\ (-\dot{\phi}_0+\Delta\omega_D-\dot{\phi}_1)\cos(\Delta\omega_D t+\phi_0'+\phi_1'-\phi_1)+(\Delta\omega_D-\dot{\phi}_1)}{\{(S/I_1)+2\sqrt{S/I_1}\ \cos(\phi_0'-\phi_1'-\phi_1)+1\}(\equiv E^2/B_1^2)} \quad . \tag{6.9}$$

Again, we observe the same limiting behaviour as (6.8) when $S/I_1 \to \infty$, viz., $\dot{\theta} \sim -\dot{\phi}_0$, as expected, and the possibility of "clicks", e.g. $\theta \to \infty$, for some (instantaneous) values of $\phi_0'-\phi_1'-\phi_1$ $(=\pm\pi)$, when $S/I_1 = 1$.

6.5 Interference Scenario (A.1-M): M($\geq$1) Unsymmetrical Interfering Signals: Equal Interference Amplitudes and Modulations

This situation is illustrated in Fig. 7, where we have the important cases of multiple adjacent interferers, where (for convenience) the amplitudes are all equal (a form of "worst case", when the total interference intensity is fixed, for example), and where different relative phase conditions are imposed (also a form of "worst-case" interference, when the relative phases are fixed, and adjusted to give maximum interference coherence). Thus, we write the conditions:

Case A.1-Ma: $B_j=B_1$, $1\leq j\leq M$; $B_0=B_0$, $j=0$; equal interference amplitudes ; (6.10a)

$\Delta\omega_j=j\Delta\omega_D$: freq. displacement proportional to interferer's spectral order; this gives equal spacing spectrally between interferers; (6.10b)

$\phi_j=\phi_1'+\phi_j$, $j\geq 1$: all interfering modulations the same; (6.10c)

$\phi_j = j\phi_1$: rel. phase is proportional to order (j) of interferer spectrally; $\phi_j=\phi_1'+j\phi_1$, $j\geq 1$; $\dot{\phi}_j=\dot{\phi}_1$, etc. (6.10d)

Direct substitution of (6.10) into (4.17), or (4.25), yields with

$$\psi_{jk} = (j-k)\Delta\omega_D t-(j-k)\phi_1 \quad , (j,k \geq 1) \ , \tag{6.11}$$

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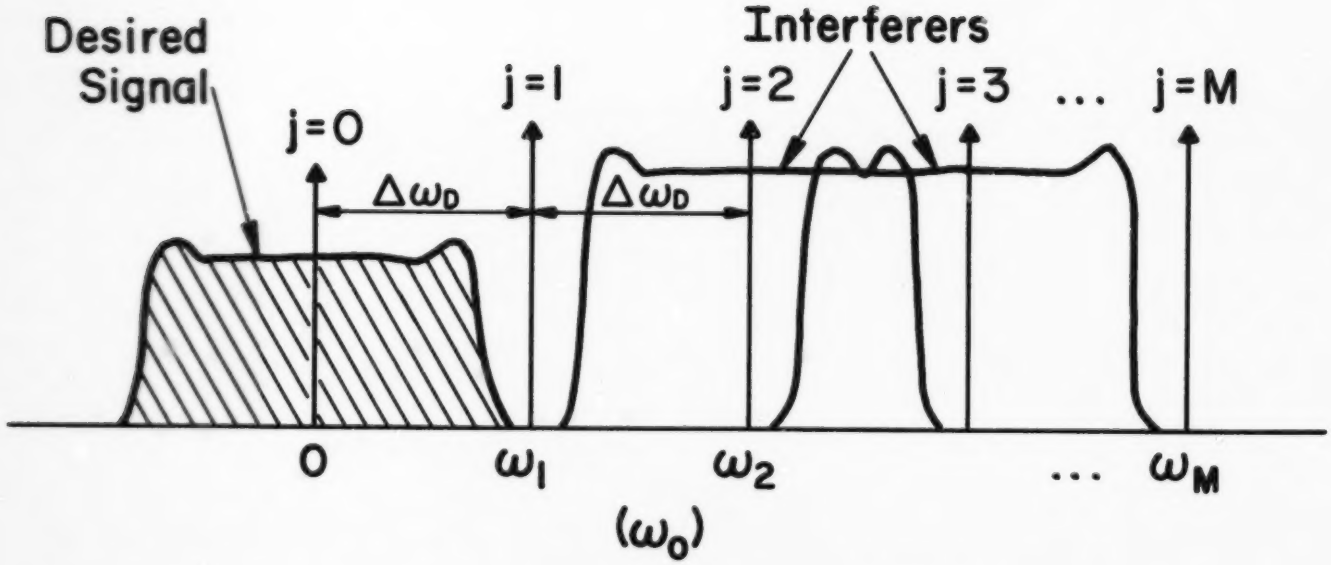

Figure 7. "Spectra" of M adjacent channel interferers, unsymmetrically located, with the desired signal.

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$$\dot{\theta}(t)\Big|_{Ma} = \Big\{ -\Phi_o + (\sqrt{S/I_M})^{-\frac{1}{2}} \sum_{k=1}^{M} (-\dot{\Phi}_o + k\Delta\omega_D - \dot{\Phi}_1) M^{-1/2} \cos[k(\Delta\omega_D t - \phi_1) - \Phi_1' + \Phi_o'] \Big.$$

$$\Big. + (S/I_M)^{-1} \sum_{jk=1}^{M} M^{-1} (j\Delta\omega_D - \dot{\Phi}_1) \cos(j-k)(\Delta\omega_D t - \phi_1) \Big\}$$

$$\Big\{ 1 + 2(S/I_M)^{-1/2} \sum_{k=1}^{M} M^{-1/2} \cos[k(\Delta\omega_D t - \phi_1) - \Phi_1' + \Phi_o'] + (S/I_M)^{-1} \sum_{jk=1} M^{-1} \cos(j-k)(\Delta\omega_D t - \phi_1) \Big\}$$

$$(\equiv E^2/B_o^2) \ . \qquad (6.12)$$

In the above,(6.12), we have used the quantity S/I_M, which we define as the average <u>independent</u> signal intensity to interference intensity ratio, S/I_M, for these equal amplitude interferers by

$$S = B_o^2/2; \quad I_M = MB_1^2/2 \quad ; \quad \therefore S/I_M = B_o^2/MB_1^2 \ . \qquad (6.13)$$

It is important to note that this intensity ratio is defined on the assumption of mutual incoherence of all signals. (Of course, when we put a condition like (6.10d) on the interferers, we are structuring the interference, introducing relative coherences which are explicit in our general formulation (and in (6.12)).) [Note that our results (6.12) always gives real values of the envelope [$E(\geq 0)$, i.e. $E^2 \geq 0$], since for $\sqrt{x} \equiv \sqrt{S/I_M}$ we have the condition on the denominator of (6.12) that

$$x - 2\sqrt{xM} + M \geq 0 \ , \text{ or } \therefore x = M > 0 \ , \qquad (6.14)$$

as required and expected.]

<u>Case A.1-Mb:</u> This is the same as Case A.1-Ma, except that in (6.10) we replace (6.10c), (6.10d) by

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$$\begin{cases} \Delta\omega_j = \Delta\omega_D, \text{ all } j\text{: all interferers are at the same displacement } (\Delta\omega_D) \\ \qquad\qquad \text{from the desired signal carrier (at } f_o). \qquad (6.15a) \\ \phi_j = \phi_1, \text{ all } j\text{: all phases are the same relative to desired signal.} \\ \qquad\qquad (6.15b) \end{cases}$$

We use (6.12) directly, to get

$$\dot{\theta}(t)\big|_{Mb} = \frac{\{-\dot{\phi}_o(S/I_M)+(S/I_M)^{1/2}M^{1/2}(-\dot{\phi}_o+\Delta\omega_D-\dot{\phi}_1)\cos(\Delta\omega_D t-\phi_1-\phi_1'+\phi_o')+M(\Delta\omega_D-\dot{\phi}_1)\}}{\{(S/I_M)+2\sqrt{M}\sqrt{S/I_M}\cos(\Delta\omega_D t-\phi_1-\phi_1'+\phi_o')+M\}\ (\equiv E^2/B_1^2 M).} \qquad (6.16)$$

Case A.1-Mc: This is the same as Case A.1-Mb, except that now all interferers are co-channel, e.g. $\Delta\omega_D=0$. From (6.16) we have at once

$$\dot{\theta}(t)\Big|_{Mc} = \frac{-\{\dot{\phi}_o(S/I_M)+\sqrt{M}\sqrt{S/I_M}\ (\dot{\phi}_o+\dot{\phi}_1)\cos(\phi_1'+\phi_1-\phi_o')+M\dot{\phi}_1\}}{\{(S/I_M)+2\sqrt{M}\sqrt{S/I_M}\cos\ (\phi_1'+\phi_1-\phi_o')+M\}(\equiv E^2/B_1^2 M)}\ . \qquad (6.17)$$

Additional simplifications occur if we impose the further condition on the interference that (all components of) the latter be the same as the desired signal waveform. Then, $\dot{\phi}_1\to\dot{\phi}_o$, $\phi_1'\to\phi_o'$ in (6.12), (6.16), (6.17). Specifically, the adjacent and cochannel cases A.1-Mb,c above reduce to

Case A.1-Md:

$$\dot{\theta}\big|_{Mb:\ adj} = \frac{-\dot{\phi}_o(S/I_M)+\sqrt{S/I_M}\ \sqrt{M}\ (\Delta\omega_D-2\dot{\phi}_o)\cos(\Delta\omega_D t-\phi_1)+M(\Delta\omega_D-\dot{\phi}_o)}{(S/I_M)+2\sqrt{S/I_M}\ \sqrt{M}\cos(\Delta\omega_D t-\phi_1)+M} \qquad (6.18)$$

and

$$\dot{\theta}\big|_{Me:coch.} = \frac{-\dot{\phi}_o\{(S/I_M+2\sqrt{M}\ \sqrt{S/I_M}\cos\ \phi_1+M\}}{(S/I_M)+2\sqrt{S/I_M}\ \sqrt{M}\cos\ \phi_1+M} = -\dot{\phi}_o\ , \qquad (6.19)$$

this last, since <u>all</u> signals are identical.

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6.6 Interference Scenario B.1-M: M($\geq$1) Symmetrical Interfering Signals: Equal Interference Amplitudes and Modulations

Here we have 2M interfering signals ($M \geq j \geq 1$; $-M \leq j \leq -1$), symmetrically spaced spectrally about the desired signal carrier, in the manner of Fig. 8. In addition, we assume equal amplitudes as above, cf. (6.10a), etc. The conditions here are explicitly:

$$B_j = B_{-j},\ 1 \leq j < M;\quad B_o = B_0,\ j=0 : \text{equal interferer amplitudes;} \tag{6.20a}$$

$$\Delta\omega_j = j\Delta\omega_D\ ,\ j<0,\ j>0: \quad \text{equal spacing between interferers;} \tag{6.20b}$$

$$\Phi_j = \Phi_1' + \phi_j,\ (j \gtrless 0): \quad \text{all interfering modulations are the same;} \tag{6.20c}$$

$$\phi_j = j\phi_1;\ (j \gtrless 0): \quad \text{all relative phases are the same cf. (6.15b).} \tag{6.20d}$$

Now, we shall indicate the substitutions in the result (6.12), to extend it to the symmetrical cases analogous to Scenarios A.1-Ma,b,c,d above. We have

Case B.1-Ma: (i). In (6.12) replace limits of Σ, e.g.

$$\sum_{j=1}^{M} \to \sum_{-M}^{M}{}^{(1)} \quad (k=0 \text{ omitted}); \qquad \sum_{j,k=1}^{M} \to \sum_{-M}^{M}{}^{(1)(1)},\ \text{all } j,k=0$$

terms omitted.

(ii). $S/I_M \to S/I_{2M} = B_o^2/2MB_1^2$;

(iii). $M \to 2M$, of course (apart from limits on Σ's). (6.21)

Carrying out (6.21) in (6.12) gives us explicitly

$$\theta(t)|_{B\text{-}Ma} = \frac{\{-\dot{\Phi}_o + (S/I_{2M})^{-\frac{1}{2}} \sum_{-M}^{M}{}^{(1)} (-\dot{\Phi}_o + k\Delta\omega_D - \dot{\Phi}_1)(2M)^{-1/2}\cos[k(\Delta\omega_D t - \phi_1) - \Phi_1' + \Phi_o'] + (S/I_{2M})^{-1} \sum_{-M}^{M}{}^{(1),(1)} (2M)^{-1}(j\Delta\omega_D - \dot{\Phi}_1)\cos(j-k)(\Delta\omega_D t - \phi_1)\}}{\{1 + 2(S/I_{2M})^{-1/2} \sum_{-M}^{M}{}^{(1)} (2M)^{-1/2}\cos[k(\Delta\omega_D t - \phi_1) - \Phi_1' + \Phi_o'] + (S/I_{2M})^{-1} \sum_{-M}^{M}{}^{(1),(1)} (\tfrac{1}{2M})\cos(j-k)(\Delta\omega_D t - \phi_1)\}(\equiv E^2/B_o^2)} \tag{6.22}$$

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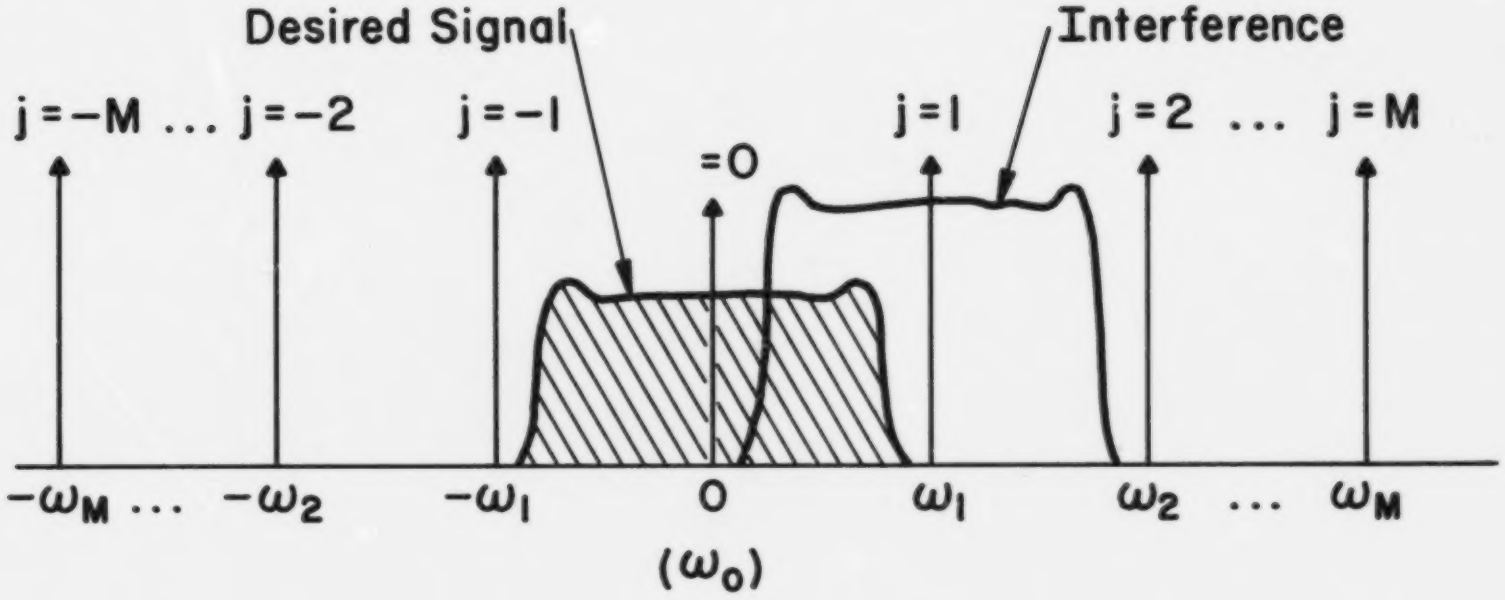

Figure 8. Same as Figure 7, with 2M (symmetrical) interferers, M on each side of the desired signal carrier frequency.

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Case B.1-Mb: Repeat conditions (i)-(iii) of (6.21) in (6.22). (6.23)

Case B.1-Mc: Repeat Case B.1-Mb with $\Delta\omega_D=0$, cf. (6.23). (6.24)

Case B.1-Md: Repeat Case B.1-Mc, now setting $\dot{\phi}_1=\dot{\phi}_o$, $\phi_1'=\phi_o'$, in (6.24), cf. (6.18), (6.19) . (6.25)

Still other possibilities may be treated similarly. For example, we have

Case C.1-Ma: (i). Set $\phi_j=j\phi_1$; $\Delta\omega_j=j\Delta\omega_D$; $B_j=B_1/|j|$; $j\gtrless 0$. This is the case above (B.1-Ma) with a regular fall-off in amplitude. (6.26)

(ii). The conditions (6.21) apply, except that (ii) therein becomes

$$S/I_M \to S/I_{2M}', \text{ with } I_{2M}' = 2\left(\sum_{j=1}^{M} B_j^2/2\right) = B_1^2 \sum_{j=1}^{M} |j|^{-1} \equiv B_1^2\sigma_M^{(1)} , \quad \text{where } \sigma_M^{(1)} \equiv \sum_{j=1}^{M} (1/j) . \tag{6.27}$$

7. NUMERICAL RESULTS AND DISCUSSION

Here we shall examine some representative numerical results, for a selection of the interference scenarios outlined in Section 6. Not only do we desire the quantitative results for the instantaneous phase, $\dot{\theta}$, and envelope E, in the above, but we wish to apply them to the various instantaneous output models outlined in Section 5 above. We shall also discuss the results in the light of the various parameter values involved, and the particular operative interference scenarios in question.

To do this, we need first to apply the specifically chosen signals of Secs. 6.1, 6.2 above to selected, illustrative scenarios. The scenarios which are selected here are:

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Case A.1, Eq. (6.8), Sec. 6.3.

Case B.1, Eq. (6.9), Sec. 6.4. (7.1)

Case B.1-Ma, Eq. (6.22), Sec. 6.5.

We employ the same type of interference and desired signal in all cases. We have specifically therefore:

Case B.1, Eq. (6.9): (M=1)

$$\dot{\theta}(t)|_{B.1} = B_o D_F\{-m(\omega_a t)(S/I_1)+\sqrt{S/I_1}(-m(\omega_a t)+\gamma/\mu_F-m(\eta\omega_a t))$$

$$\frac{\cos(\omega_a\gamma t+\phi_o'-\phi_1'-\phi_1)+(\gamma/\mu_F-m(\eta\omega_a t))\}}{\{(S/I_1)+2\sqrt{S/I_1}\cos(\gamma\omega_a t+\hat{\phi}_o'-\phi_1'-\phi_1)+1\}}\ , \qquad (7.2)$$

with ϕ_o' = (6.1a), (6.2b); ϕ_1' = (6.5b), (6.6b), and $m(\omega_a t)$, $m(\eta\omega_a t)$, from (6.1a), (6.2a), (6.5a), respectively. The parameters (γ,μ_F, etc.) are given in (6.3).

Case A.1, Eq. (6.8); (M=1):

$$\dot{\theta}(t)|_{A.1} = \text{Eq. (7.2) with } \Delta\omega_D=0, \text{ or } \gamma=0. \qquad (7.3)$$

Case B.1-Ma, Eq. (6.22): (M≥1):

This is the case involving two or more (M≥1) symmetrical interferers, for which Eq. (6.22) reduces, for the specific signal forms developed in Section 6.1, 6.2, to

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$$\dot{\theta}(t)\big|_{B.1\text{-}Ma} = B_o D_F \Bigg\{ -m(\omega_a t)(S/I_{2M}) + (S/I_{2M})^{\frac{1}{2}} \sum_{k=-M}^{M} (\text{omit } k=0)(2M)^{-1/2}$$

$$\left[\left(-m(\omega_a t) + \frac{k\gamma}{\mu_F} - m(\eta\omega_a t)\right) \cos\left[k(\gamma\omega_a t - \phi_1 + \phi_o' - \phi_1'\right]\right]$$

$$+ \frac{1}{2M} \sum_{j,k=-M}^{M} (\text{exclude } j,k=0, \text{ each+both}) \left[j\gamma/\mu_F - m(\eta\omega_a t)\right] \cos(j-k)(\gamma\omega_a t - \phi_1) \Bigg\}$$

$$\Bigg\{ (S/I_{2M}) + 2\sqrt{S/I_{2M}} \sum_{-M}^{M} (\text{omit } k=0)(2M)^{-1/2} \cos\left[k(\gamma\omega_a t - \phi_1) + \phi_o' - \phi_1'\right]$$

$$+ \frac{1}{2M} \sum_{-M}^{M} (\text{omit } j,k=0, \text{ each+both}) \cos(j-k)(\gamma\omega_a t - \phi_1) \Bigg\} (\equiv E^2/2B_1^2 M) \ ,$$

where $M \geq 1$. (7.4)

Our first example (Case B.1, equation 7.2) is given in figure 9. Here the signal-to-interference ratio is -10 dB and two cases are shown; modulation index $\mu_F = 1$ and $\gamma = 0.5$ and $\mu_F = 10$ and $\gamma = 5$. The desired signal is $\cos(\omega_a t)$ and the interfering signal is $\cos(\eta\omega_a t)$ with $\eta = \pi/2$. Note the output waveform follows the interference (with the appropriate π phase shift). For the two cases the spectral "picture" is the same, i.e., the interfering carrier is half way between ω_o and $\omega_o + \mu_F\omega_a$. Note, however, that the "broadband" FM case ($\mu_F = 10$) results in high frequency oscillations riding the "basic" output waveform. Figure 10 is the same situation as figure 9, except now the signal-to-interference ratio is 10 dB. The results are the same except now, of course, the output waveform follows the desired signal.

Figure 11 shows the baseband output waveform for a signal-to-interference ratio of 2 dB showing the difference between one interfering signal ($\mu_F = 1$, $\gamma = 1$, equation 7.2) and two interfering signals of the same power ($\mu_F = 1$, γ 1, equation 7.4). Note that is (7.4), S/I_{2M} is defined using independent interferers and for figure 11, $\phi_1 = 0$ so the two interferers add coherently. This means an appropriate adjustment in S/I_{2M} must be made to obtain a true comparison. That is, an S/I_{2M} of 1 dB ($\mu = 1$) in (7.4) corresponds to and S/I of 2 dB in (7.2). Note from figure 11 that the results are similar, except the single interferer produces a much larger "click" (the "spike" going down to about -4) than do the two coherent interferers.

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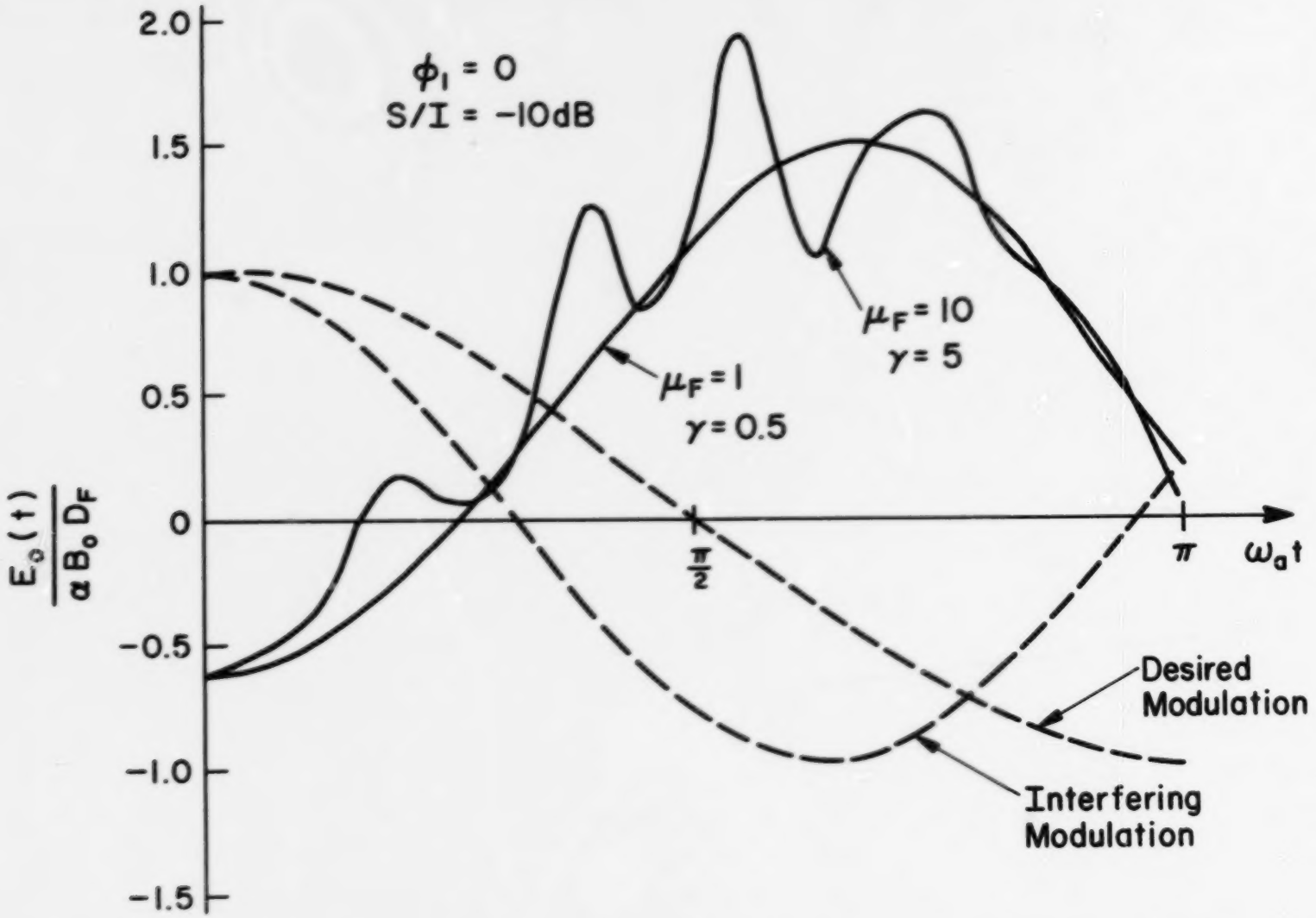

Figure 9. Output baseband waveform $E_o(t)$ for a signal-to-interference ratio of -10 dB for the two cases $\mu_F = 1$, $\gamma = 0.5$ and $\mu_F = 10$, $\gamma = 5$ (eq 7.2). The desired modulation is cos $(\omega_a t)$ and the interfering modulation is $\cos(n\omega_a t)$ with $n = \pi/2$. The phase difference $\phi_1 = 1$. Class I, the ideal receiver (eq. 5.2), Case B.1.

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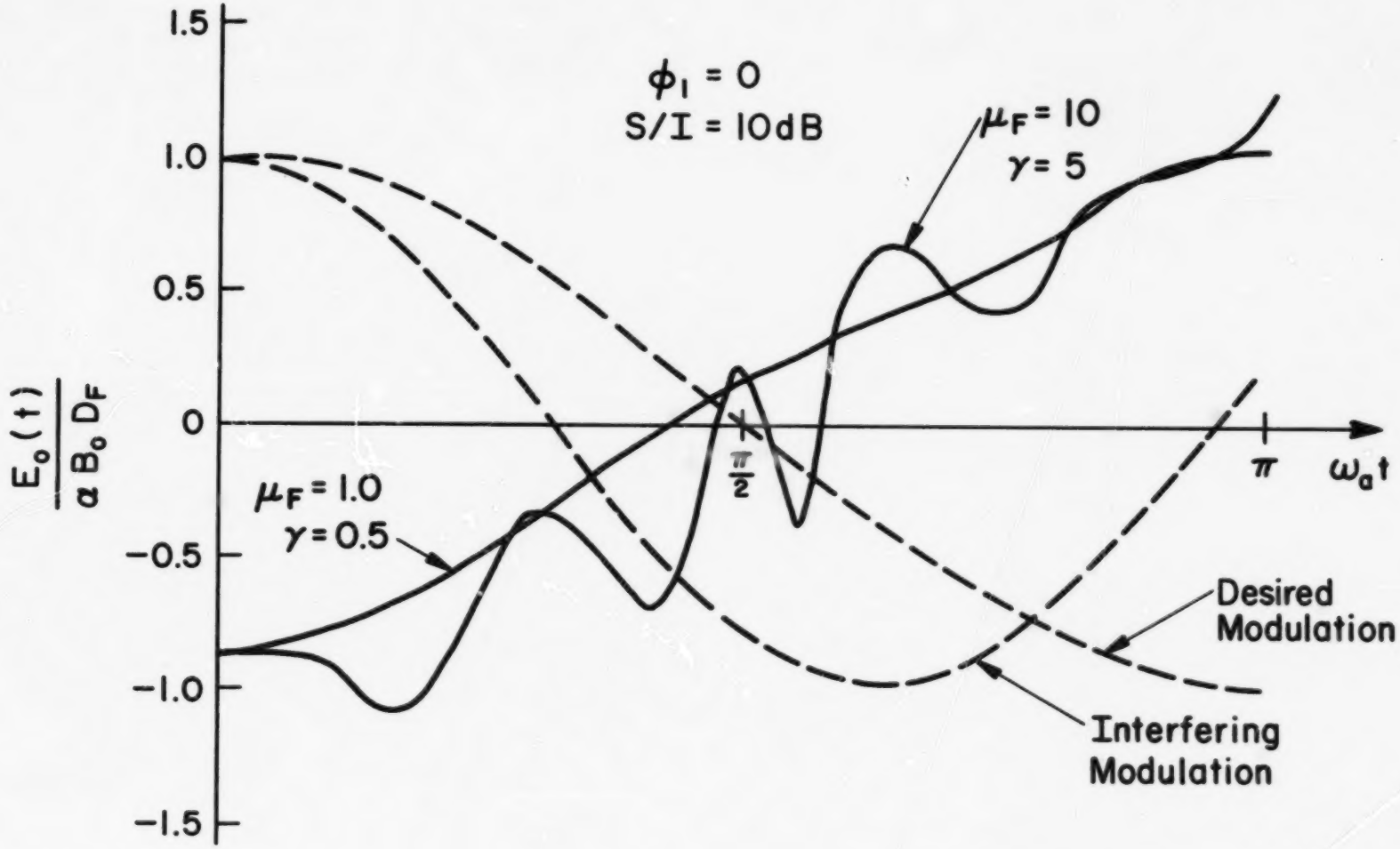

Figure 10. Sames as figure 9, but the signal-to-interference ratio is 10 dB.

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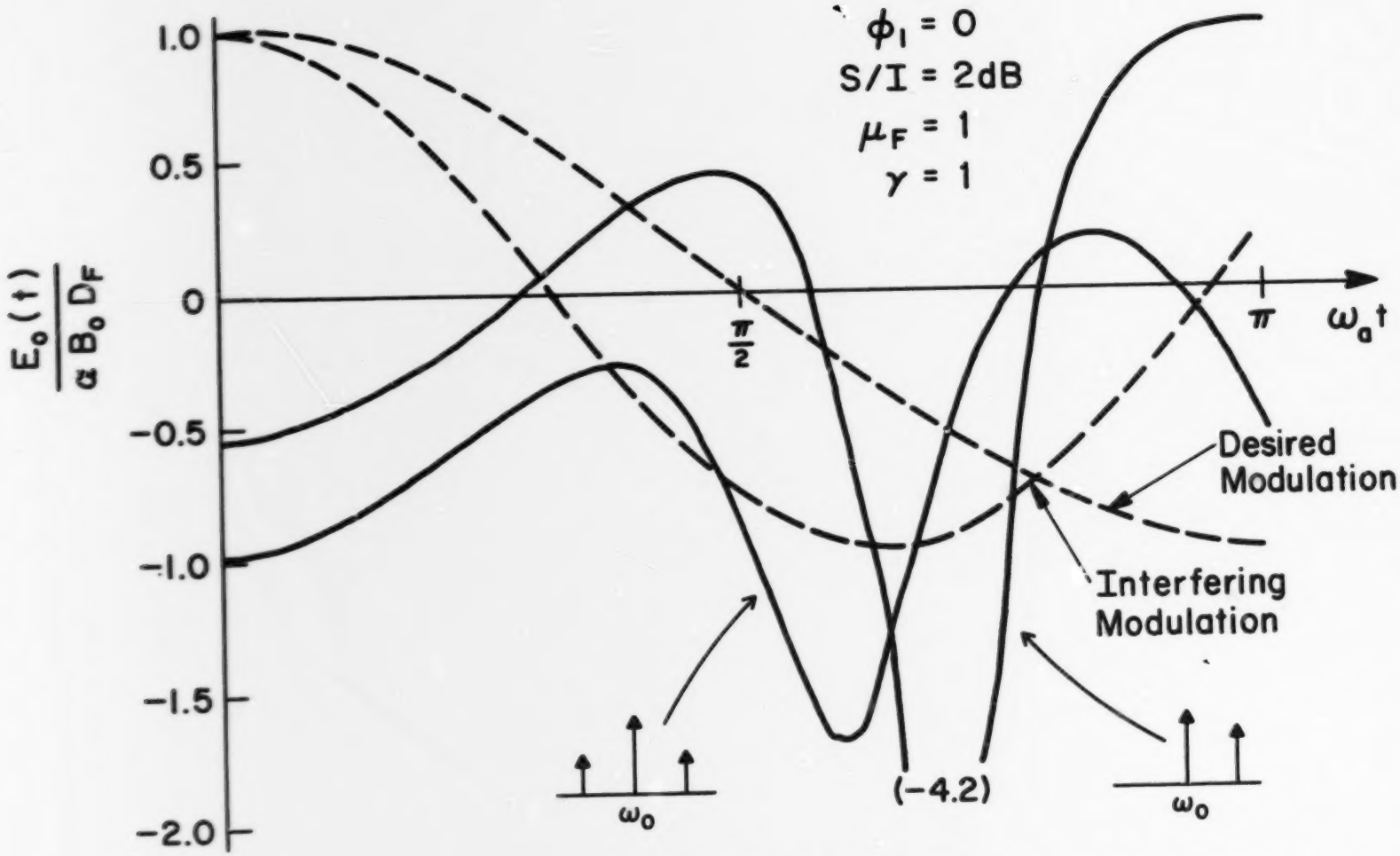

Figure 11. Output baseband waveform $E_0(t)$ for a signal-to-interference ratio of 2 dB showing the difference between one interfering signal (eq. 7.2) versus two symmetrically placed interferers (eq. 7.4) ($\phi_1 = 0$, so all signals are in phase), Cases B.1 and B.1-Ma.

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Figure 12 shows the effect of varying the spacing between the desired carrier and one interferring carrier (7.2) for the case μ_F = 10 and S/I = 10 dB. For γ = 0; Case A.1, cochannel interference, the output reasonably follows the desired output. As the interfering carrier is removed from the desired carrier, high frequency oscillations arise depending on the spacing and on the interfering and desired modulations as shown. Remember there is no front end filtering.

Figure 13 is as in figure 12 except now the situation is μ_F = 1, and M = 5 in equation 7.4. (10 symmetrical interferers). The output is given for γ 0.1, 0.5, and 1.0. Increasing the spacing further (γ = 2, say) results in higher frequency oscillations as in figure 12, Cases B.1-Ma, A.1.

The above figures show a small sample of the many situations possible to analyze using the results obtained here. Also, it is easy to obtain the spectrum of the output (and thereby determine the "usual" distortion factors) by means of the Fast Fourier Transform.

8. GENERAL RESULTS AND CONCLUSIONS

In this paper we have presented a general analysis of nonideal FM receivers where the interference with the desired FM signal consists of one or more similar FM signals, which may appear co-channel or off-tune from the desired signal. Typical non-ideal limiter and discriminator structures are specifically included, with the only simplification here being that of ideal, i.e., very broad-band-front-end stages (antenna, RFx IF), so that no distortion of the angle-modulated waves is introduced. Various signal levels, frequency displacements, and relative phases are employed, to outline typical interference scenarios, which are discussed in Sections 6, 7 above. In fact, the limitations of distortionless front-end stages has been removed in a subsequent analysis (6). The basic inputs to the non-linear portions of the receiver are structurally unchanged with the addition of amplitude modulation produced by the "scanning" of the front-end response by the angle modulation. The resulting signal waveforms, of course, are correspondingly modified.

Apart from the present idealization of the linear (i.e., front-end) stages of the typical receiver, our approach is fully general, and is analytically and computationally much more direct and easier to apply than conventional treatments, which employ direct harmonic analyses from the very start. (This

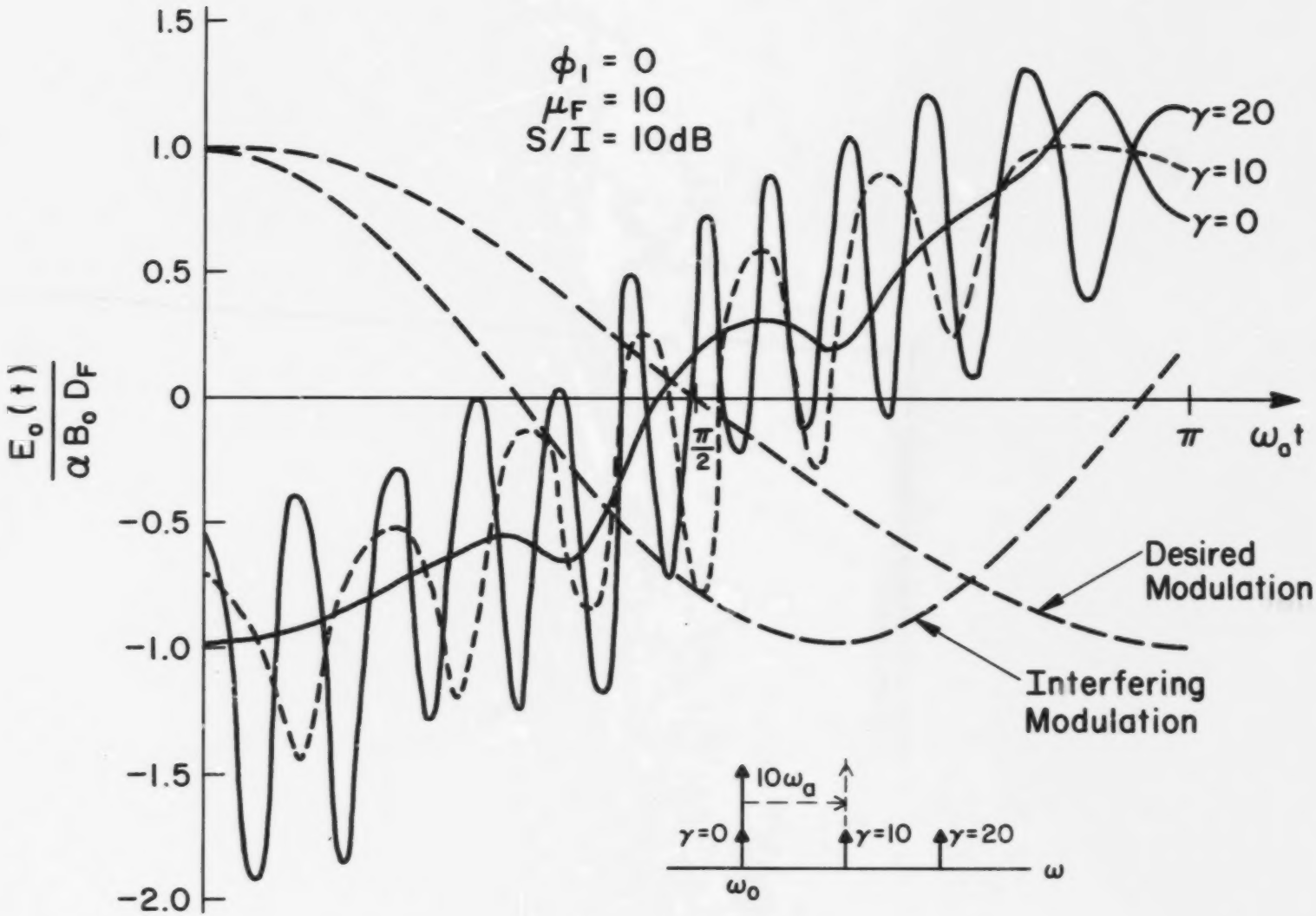

Figure 12. Output baseband waveform $E_o(t)$ for signal-to-interference ratio of 10 dB and a modulation index μ_F of 10, on interfering signal, for various displacements (γ) of the interfering signal from the desired signal (eq. 7.2), cases A.1, B.1.

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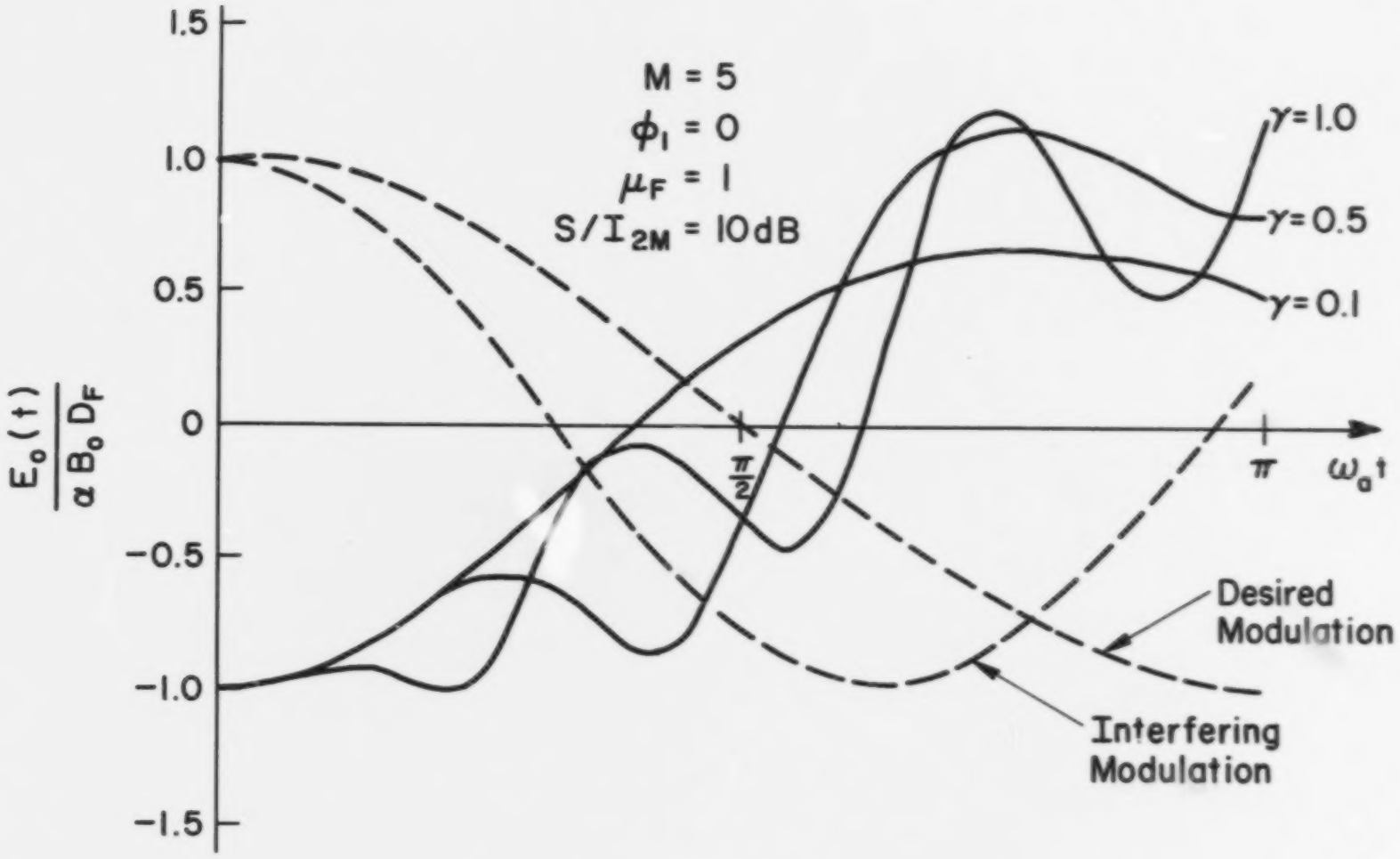

Figure 13. Output baseband waveform $E_o(t)$ for 10 symmetrical interferers (eq. 7.4) for a signal-to-interference ratio of 10 dB for various spacings of interfering signals (γ = 0.1, 0.5, 1) for μ_F = 1 and ϕ_1 = 0, Case b.1-Ma.

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accounts, not very surprisingly, for the conspicious absence of Bessel functions in the analysis, since our treatment is an instantaneous formulation for the output.)

A second critical feature of the analysis is that it permits the direct representation of particular scenarios, which can then be studied in detail, as well as general structures, characteristic of the broad classes of determininstic interference scenarios encountered practically. Moreover, because of the construction of the associated software used to obtain the numerical results of Section 7 above, we can also obtain any other desired waveforms, for other sets of parameters and interfering signal configurations.

Various specific questions, as well, can be answered from the particular numerical results obtained from this general, "instantaneous" treatment. For example, we see the effect of changing the frequency spacing between the desired signal ($f_o = 0$) and the one (or more) interfering signals, ($\Delta\omega_j > 0$). In figure 12, where $\mu_F = 10$, and $M = 1$ (i.e., there is one "off-tune" interferer), when the frequency spacing ($\sim \gamma$) is changed. The major effect is to intorduce a progressively greater "ripple" or "beat" in the output waveform as the spacing is increased. Similarly, in Figure 13 (for $\mu_F = 1$, and 10 symmertical interferers), the effect of increased frequency spacing is again to "modulate" or show "beats" in the output, with this effect becomming greater as the separation (γ) is made larger.

As another example, we may compare the effects of one (off-tune) interferer, with two similar, symmetrical interferers, (of same total intensity). The unsymmetrical interferer introduces a rather drastic variation of the output waveform in figure 11, while the symmetrical pair produces a much less noticeable distortion of the output. (The particular curves shown in figure 11, are for coherent addition ($\phi = 0$) of the interferers, e.g., $(S/2B_1)^2$). Similar effects may be expected for other parameter choices. Generally, symmetrical interferers (with some degree of phase coherence) produce less distortion than the unsymmetrical (i.e., one-sided) interfering signals.

It is clear, of course, that these above general classes of interference are (statistically) highly non-Gaussian, and it is for this reason, and the fact that one or two (or a few more) FM signals provide many types of typical interference environments, that we have undertaken here to develop a direct, deterministic, analytic theory. Such a theory, however, necessarily has limitations: (i) arbitrary choices of relative signal phase, (ii) arbitrary

amplitude levels; and (iii) similar ad hoc selection of the other parameters, rather than an a priori randomized selection of such values. The result is to a certain degree a "special case" or "limiting case" analysis rather than a fully randomized treatment which reflects the receiver's a priori uncertainty as to exact parameter values, which is the essence and power of the classical statistical approach. Nevertheless, our direct deterministic analysis, plus the associated computational programs, does provide a reasonable "spectrum" of typical results, which to date are not available statistically, and which do show the characteristic waveform modifications which are produced by the higher nonlinear receiver operations embodied in these nonideal limiter and discriminator functions of typical FM receivers.

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APPENDIX: COMPUTER SOFTWARE

In this appendix, we list the computer programs, along with sample outputs, which were used to obtain the numerical results presented in this report, and which then, can be used to obtain a wide range of similar results. The programs are designed to be versatile and essentially self explanatory via the comment statements which relate the computed results to the various equations, etc., in the report.

The first program listing given is titled FM2. This program is for a single interferer, either cochannel or adjacent channel, depending on γ, and gives the results of equation 7.2. The sample output given after the program listing is the $\gamma = 10$ curve of figure 12. All the figures in the report are for the ideal receiver (5.2 of the report) and is the column titled "THETA". The program also computes the output waveforms for the other receiver types -- (5.3) is given by the column "E2", (5.4) by column "E3" and (5.5) by column "E4". In (5.4) and (5.5) the parameters b^2 and ν are given the arbitrary value 1. Of course, for actual situations, appropriate values of b^2 and ν would need to assigned. The program listed, FM2, computes the output waveform at 40 points between 0 and π ($\omega_a t$): The number of points needed depends on the number of oscillators in the output waveform and the number of points used is set by the parameter "F". The mean and mean-square values of all the output waveforms are obtained via "SUM1" through "SUM14" and are printed out as the last two lines of the output, the first line being the mean values and the second the mean-square values. Also given are the desired modulation and the interfering modulations in the last two columns -- "MOD" and "INT".

The other program given is titled FM2M and covers the situation of 2M symmetrical interferers, equation 7.4. The number of interferers is set by the parameter "NN" and the total number is given by 2NN, and is 10 in the example listed. The various output waveforms and their mean and mean-square values are given as in the program FM2 above. The sample output given along with the program listing is for the case $\phi = 0$, $\mu_F = 1$, $\gamma = 1.5$, and S/I = 10 dB. Note that the output waveforms are computed at 20 points, and as for FM2, the number of points is set by the parameter "F".

For the two programs as listed, FM2 (3 signal-to-interference ratios, 4 values of γ, and 2 values of μ_F, or 24 total situations) required 4.7 seconds of execution time on a CYBER 170/750 computer, and FM2M (10 signal-to-interference ratios, 3 values of γ, and 2 values of μ_F, 10 symetrical interferers, or 60 situations) required 19 seconds of execution time.

```
      PROGRAM FM2(INPUT,OUTPUT)
C     THIS PROGRAM COMPUTES THE BASEBAND FM OUTPUTS, MIDDLETONS
C     FM REPORT 1, EQUATION 7.2 (OR 6.9), FOR THE DESIRED MODULATION
C     COS(WAT) (6.1A), AND INTERFERING MODULATION COS(ETA*WAT).
C     ALSO GIVEN VIA SUM2 THROUGH SUM14 ARE ESTIMATES OF THE
C     MEAN AND MEAN SQUARE VALUES OF THE VARIOUS WAVEFORMS.
      DIMENSION WAT(41),YY(41),THETA(41),E1(41),E2(41),E3(41),
     1E4(41),GAMM(4),DB(3)
      DIMENSION YYI(41),AYY(41),AYYI(41)
      DATA GAMM/0.,1.,2.,3./
      DATA DB/-10.,2.,10./
      PI=3.14159265
      ETA=PI/2.
      F=1./40.
      PRINT 6
    6 FORMAT(1H1)
      PHI=0.
      DO 80 J=1,2
      AMUF=10.**(J-1)
      DO 70 K=1,4
      GAM=AMUF*GAMM(K)
      DO 60 L=1,3
      SOIDB=DB(L)
      SOI=10.**(SOIDB/10.)
      SUM1=0. $ SUM2=0. $ SUM3=0. $ SUM4=0. $ SUM5=0. $ SUM6=0.
      SUM7=0. $ SUM8=0. $ SUM9=0. $ SUM10=0. $ SUM11=0. $ SUM12=0.
      SUM13=0. $ SUM14=0.
      DO 50 M=1,41
      WAT(M)=(PI/40.)*(M-1)
      YY(M)=COS(WAT(M))
      YYI(M)=COS(ETA*WAT(M))
      AYY(M)=AMUF*SIN(WAT(M))
      AYYI(M)=AMUF*SIN(ETA*WAT(M))/ETA
      TOP=-YY(M)*SOI+(SOI**0.5)*(-YY(M)-YYI(M)+GAM/AMUF)*COS(GAM*WAT(M)
     1+AYY(M)-AYYI(M)-PHI)+(GAM/AMUF-YYI(M))
      BOT=SOI+2.*(SOI**0.5)*COS(GAM*WAT(M)+AYY(M)-AYYI(M)-PHI)+1.0
      THETA(M)=TOP/BOT
C     THETA IS THETADOT/BOOF (EQ. 7.2).
C     ALSO, E0=ALPHA*THETADOT (EQ. 5.2, IDEAL RECEIVER).
      E1(M)=BOT**0.5
C     E1 IS E/B1, EQ. 7.2 OR 6.9, DENOMINATOR.
      E2(M)=TOP/E1(M)
C     E2 IS E0/ALPHA, EQ. 5.3, (NO LIMITING, IDEAL DISCRIM.).
      E3(M)=(TOP/BOT)/(1.+(ABS(TOP/BOT))**4.)
C     E3 IS E0/ALPHA, EQ. 5.4, FOR B SQUARED=1. AND MU=2..
      E4(M)=E3(M)*E1(M)
C     E4 IS E0/ALPHA, EQ. 5.5.
      IF(M.EQ.1) GO TO 50
      SUM1=SUM2+F*(THETA(M-1)+THETA(M))/2.
      SUM2=SUM2+F*((THETA(M-1)+THETA(M))/2.)**2.
      SUM3=SUM3+F*(E1(M-1)+E1(M))/2.
      SUM4=SUM4+F*((E1(M-1)+E1(M))/2.)**2.
      SUM5=SUM5+F*(E2(M-1)+E2(M))/2.
      SUM6=SUM6+F*((E2(M-1)+E2(M))/2.)**2.
      SUM7=SUM7+F*(E3(M-1)+E3(M))/2.
      SUM8=SUM8+F*((E3(M-1)+E3(M))/2.)**2.
      SUM9=SUM9+F*(E4(M-1)+E4(M))/2.
```


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```
      SUM10=SUM10+F*((E4(M-1)+E4(M))/2.)**2.
      SUM11=SUM11+F*(YY(M-1)+YY(M))/2.
      SUM12=SUM12+F*((YY(M-1)+YY(M))/2.)**2.
      SUM13=SUM13+F*(YYI(M-1)+YYI(M))/2.
      SUM14=SUM14+F*((YYI(M-1)+YYI(M))/2.)**2.
   50 CONTINUE
      PRINT 7, PHI,AMUF,GAM,SOICB
    7 FORMAT(5X,*PHI=*,F3.1,2X,*MUF=*,F4.1,2X,*GAMMA=*,F4.1,
     12X,*S/I=*,F5.1,/)
      PRINT 8
    8 FORMAT(10X,*WAT*,9X,*THETA*,9X,*E1*,11X,*E2*,11X,*E3*,
     111X,*E4*,9X,*MOD*,10X,*INT*)
      DO 40 JJ=1,41
       PRINT 9, WAT(JJ),THETA(JJ),E1(JJ),E2(JJ),E3(JJ),E4(JJ),YY(JJ),
     1YYI(JJ)
   40 CONTINUE
    9 FORMAT(5X,8(1PE12.5,1X))
      PRINT 10
   10 FORMAT(5X,*----------------------------------------------------------
     1----------------------------------------*)
      PRINT 11,SUM1,SUM3,SUM5,SUM7,SUM9,SUM11,SUM13
   11 FORMAT(18X,7(1PE12.5,1X))
      PRINT11,SUM2,SUM4,SUM6,SUM8,SUM10,SUM12,SUM14
      PRINT 12
   12 FORMAT(5X,*==========================================================
     1========================================*,//)
      PRINT 6
   60 CONTINUE
   70 CONTINUE
   80 CONTINUE
      END
```


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PHI=0.0 MUF=10.0 GAMMA=10.0 S/I= 10.0

MAT	THETA	E1	E2	E3	E4	MOD	INT
0.	-7.59747E-01	4.16228E+00	-3.16228E+00	-5.69877E-01	-2.37199E+00	1.00000E+00	1.00000E+00
7.85398E-02	-7.86918E-01	3.93279E+00	-3.09478E+00	-5.68805E-01	-2.23699E+00	9.96917E-01	9.92400E-01
1.57080E-01	-8.97417E-01	3.30761E+00	-2.96831E+00	-5.44351E-01	-1.80050E+00	9.87688E-01	9.69714E-01
2.35619E-01	-1.18496E+00	2.52754E+00	-2.99503E+00	-3.98764E-01	-1.00789E+00	9.72370E-01	9.32288E-01
3.14159E-01	-1.44222E+00	2.16634E+00	-3.12433E+00	-2.70769E-01	-5.86576E-01	9.51057E-01	8.80690E-01
3.92699E-01	-1.06078E+00	2.68642E+00	-2.84971E+00	-4.68087E-01	-1.25748E+00	9.23880E-01	8.15705E-01
4.71239E-01	-7.28256E-01	3.54140E+00	-2.57905E+00	-5.68382E-01	-2.01287E+00	8.91007E-01	7.38320E-01
5.49779E-01	-5.72898E-01	4.10190E+00	-2.34997E+00	-5.17185E-01	-2.12145E+00	8.52640E-01	6.49712E-01
6.28319E-01	-5.26565E-01	4.04198E+00	-2.17837E+00	-4.88973E-01	-1.97642E+00	8.09017E-01	5.51228E-01
7.06858E-01	-6.42615E-01	3.31094E+00	-2.12766E+00	-5.48994E-01	-1.81769E+00	7.60406E-01	4.44365E-01
7.85398E-01	-1.15060E+00	2.33944E+00	-2.69176E+00	-4.17997E-01	-9.77879E-01	7.07107E-01	3.30748E-01
8.63938E-01	-1.05165E+00	2.40219E+00	-2.52627E+00	-4.73041E-01	-1.13634E+00	6.49448E-01	2.12102E-01
9.42478E-01	-3.94712E-01	3.48238E+00	-1.37454E+00	-3.85358E-01	-1.34196E+00	5.87785E-01	9.02328E-02
1.02102E+00	-1.51527E-01	4.14822E+00	-6.28566E-01	-1.51447E-01	-6.28235E-01	5.22499E-01	-3.30084E-02
1.09956E+00	-1.61315E-01	3.76069E+00	-6.06653E-01	-1.61205E-01	-6.06243E-01	4.53991E-01	-1.55748E-01
1.17810E+00	-6.80011E-01	2.57391E+00	-1.75029E+00	-5.60220E-01	-1.44196E+00	3.82683E-01	-2.76120E-01
1.25664E+00	-8.52917E-01	2.34304E+00	-1.93842E+00	-5.57750E-01	-1.30683E+00	3.09017E-01	-3.92294E-01
1.33518E+00	2.54073E-02	3.58115E+00	9.09873E-02	2.54073E-02	9.09872E-02	2.33445E-01	-5.02506E-01
1.41372E+00	2.66005E-01	4.15878E+00	1.10626E+00	2.64680E-01	1.10075E+00	1.56434E-01	-6.05079E-01
1.49226E+00	1.16764E-01	3.39625E+00	3.96561E-01	1.16743E-01	3.96488E-01	7.84591E-02	-6.98454E-01
1.57080E+00	-7.50633E-01	2.20993E+00	-1.65884E+00	-5.69751E-01	-1.25911E+00	1.79489E-09	-7.81212E-01
1.64934E+00	1.00482E-02	2.89058E+00	2.90450E-02	1.00482E-02	2.90450E-02	-7.84591E-02	-8.52095E-01
1.72788E+00	5.44954E-01	4.02011E+00	2.19077E+00	5.00787E-01	2.01322E+00	-1.56434E-01	-9.10025E-01
1.80642E+00	5.98452E-01	3.95374E+00	2.36612E+00	5.30417E-01	2.09713E+00	-2.33445E-01	-9.54122E-01
1.88496E+00	1.92341E-01	2.81057E+00	5.40587E-01	1.92078E-01	5.39848E-01	-3.09017E-01	-9.83716E-01
1.96350E+00	-3.08977E-01	2.20196E+00	-6.80356E-01	-3.06186E-01	-6.74211E-01	-3.82683E-01	-9.98356E-01
2.04204E+00	5.58042E-01	3.22524E+00	1.79982E+00	5.08709E-01	1.64071E+00	-4.53990E-01	-9.97821E-01
2.12058E+00	8.56498E-01	4.07364E+00	3.48906E+00	5.56836E-01	2.26835E+00	-5.22499E-01	-9.82118E-01
2.19911E+00	8.89228E-01	4.01644E+00	3.57153E+00	5.47133E-01	2.19753E+00	-5.87785E-01	-9.51485E-01
2.27765E+00	7.37900E-01	3.23638E+00	2.38813E+00	5.69158E-01	1.84201E+00	-6.49448E-01	-9.06389E-01
2.35619E+00	3.59830E-01	2.36504E+00	8.51012E-01	3.53897E-01	8.36980E-01	-7.07107E-01	-8.47516E-01
2.43473E+00	3.47052E-01	2.22730E+00	7.72988E-01	3.42089E-01	7.61934E-01	-7.60406E-01	-7.75759E-01
2.51327E+00	7.43650E-01	2.79992E+00	2.08216E+00	5.69486E-01	1.59452E+00	-8.09017E-01	-6.92210E-01
2.59181E+00	9.39538E-01	3.42578E+00	3.21865E+00	5.28063E-01	1.80903E+00	-8.52640E-01	-5.98139E-01
2.67035E+00	1.00949E+00	3.84853E+00	3.88506E+00	4.95211E-01	1.90583E+00	-8.91007E-01	-4.94976E-01
2.74889E+00	1.02876E+00	4.06597E+00	4.18291E+00	4.85241E-01	1.97297E+00	-9.23880E-01	-3.84289E-01
2.82743E+00	1.02654E+00	4.14697E+00	4.25702E+00	4.86407E-01	2.01712E+00	-9.51057E-01	-2.67760E-01
2.90597E+00	1.01436E+00	4.16225E+00	4.22204E+00	4.92720E-01	2.05082E+00	-9.72370E-01	-1.47161E-01
2.98451E+00	9.96481E-01	4.16016E+00	4.14552E+00	5.01753E-01	2.08737E+00	-9.87688E-01	-2.43245E-02
3.06305E+00	9.73913E-01	4.16129E+00	4.05273E+00	5.12676E-01	2.13339E+00	-9.96917E-01	9.88813E-02
3.14159E+00	9.47061E-01	4.16021E+00	3.93997E+00	5.24841E-01	2.18345E+00	-1.00000E+00	2.20584E-01
	5.59565E-01	3.34915E+00	2.97373E-01	1.52439E-02	1.77528E-01	1.14206E-09	-1.97400E-01
	5.58616E-01	1.16133E+01	6.48044E+00	1.89336E-01	2.34753E+00	4.99229E-01	4.76328E-01

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```
      PROGRAM FM2M(INPUT,OUTPUT)
C     THIS PROGRAM COMPUTES THE BASEBAND OUTPUT ETC. AS PER
C     PROGRAM FM2. BUT THERE ARE NOW 2M SYMETRICAL IDENTICAL,
C     EQUALLY SPACED ACCORDING TO PARAMETER GAMMA, INTERFERERS.
      DIMENSION WAT(21),YY(21),THETA(21),E1(21),E2(21),E3(21),
     1E4(21),GAMM(3),DB(10)
      DIMENSION YYI(21),AYY(21),AYYI(21)
      DATA GAMM/1.5,2.0,3.0/
      DATA DB/-20.,-15.,-10.,-5.,-2.,2.,5.,10.,15.,20./
      PI=3.14159265
      ETA=PI/2.
      F=1./20.
      PHI=0.
      NN=5
C     2*NN IS THE NUMBER OF SYMMETRICAL INTERFERERS.
      PRINT 6
    6 FORMAT(1H1)
      DO 80 J=1,2
      AMUF=10.**(J-1)
      DO 70 K=1,3
      GAM=AMUF*GAMM(K)
      DO 60 L=1,10
      SOIDB=DB(L)
      SOI=10.**(SOIDB/10.)
      SUM1=0. $ SUM2=0. $ SUM3=0. $ SUM4=0. $ SUM5=0. $ SUM6=0.
      SUM7=0. $ SUM8=0. $ SUM9=0. $ SUM10=0. $ SUM11=0. $ SUM12=0.
      SUM13=0. $ SUM14=0.
      DO 50 M=1,21
      WAT(M)=(PI/20.)*(M-1)
      YY(M)=COS(WAT(M))
      YYI(M)=COS(ETA*WAT(M))
      AYY(M)=AMUF*SIN(WAT(M))
      AYYI(M)=AMUF*SIN(ETA*WAT(M))/ETA
      S1=0.
      NNN=2*NN+1
      DO 31 N1=1,NNN
      Z1=(N1-1)-NN
      IF(Z1.EQ.0.) GO TO 31
      S1=S1+(-YY(M)-YYI(M)+Z1*GAM/AMUF)*
     1COS(Z1*(GAM*WAT(M)-PHI)+AYY(M)-AYYI(M))
   31 CONTINUE
      S2=0.
      DO 32 N2=1,NNN
      Z2=(N2-1)-NN
      IF(Z2.EQ.0.) GO TO 32
      DO 33 N3=1,NNN
      Z3=(N3-1)-NN
      IF(Z3.EQ.0.) GO TO 33
      S2=S2+(Z2*GAM/AMUF-YYI(M))*COS((Z2-Z3)*(GAM*WAT(M)-PHI))
   33 CONTINUE
   32 CONTINUE
      S3=0.
      DO 36 N4=1,NNN
      Z4=(N4-1)-NN
      IF(Z4.EQ.0.) GO TO 36
      S3=S3+COS(Z4*(GAM*WAT(M)-PHI)+AYY(M)-AYYI(M))
   36 CONTINUE
```


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```
      S4=0.
      DO 37 N5=1,NNN
      Z5=(N5-1)-NN
      IF(Z5.EQ.0.) GO TO 37
      DO 38 N6=1,NNN
      Z6=(N6-1)-NN
      IF(Z6.EQ.0.) GO TO 38
      S4=S4+COS((Z5-Z6)*(GAM*WAT(M)-PHI))
   38 CONTINUE
   37 CONTINUE
      TOP=-YY(M)*SOI+(SOI**0.5)*(1./((2.*NN)**0.5))*S1+S2/(2.*NN)
      BOT=SOI+2.*(SOI**0.5)*(1./((2.*NN)**0.5))*S3+S4/(2.*NN)
      THETA(M)=TOP/BOT
C     THETA IS THETADOT/BODF (EQ.7.4).
C     ALSO, E0=ALPHA*THETADOT (EQ.5.2, IDEAL RECEIVER).
      E1(M)=BOT**0.5
C     E1 IS SQUAREROOT OF E**2./(2.*M*B1**2.), EQ. 7.4, DENOMINATOR.
      E2(M)=TOP/E1(M)
C     E2 IS E0/ALPHA, EQ. 5.3, (NO LIMITING, IDEAL DISCRIM.).
      E3(M)=(TOP/BOT)/(1.+(ABS(TOP/BOT))**4.)
C     E3 IS E0/ALPHA, EQ. 5.4, FOR B SQUARED=1. AND MU=2..
      E4(M)=E3(M)*E1(M)
C     E4 IS E0/ALPHA, EQ.5.5.
      IF(M.EQ.1) GO TO 50
      SUM1=SUM2+F*(THETA(M-1)+THETA(M))/2.
      SUM2=SUM2+F*((THETA(M-1)+THETA(M))/2.)**2.
      SUM3=SUM3+F*(E1(M-1)+E1(M))/2.
      SUM4=SUM4+F*((E1(M-1)+E1(M))/2.)**2.
      SUM5=SUM5+F*(E2(M-1)+E2(M))/2.
      SUM6=SUM6+F*((E2(M-1)+E2(M))/2.)**2.
      SUM7=SUM7+F*(E3(M-1)+E3(M))/2.
      SUM8=SUM8+F*((E3(M-1)+E3(M))/2.)**2.
      SUM9=SUM9+F*(E4(M-1)+E4(M))/2.
      SUM10=SUM10+F*((E4(M-1)+E4(M))/2.)**2.
      SUM11=SUM11+F*(YY(M-1)+YY(M))/2.
      SUM12=SUM12+F*((YY(M-1)+YY(M))/2.)**2.
      SUM13=SUM13+F*(YYI(M-1)+YYI(M))/2.
      SUM14=SUM14+F*((YYI(M-1)+YYI(M))/2.)**2.
   50 CONTINUE
      PRINT 7, PHI,AMUF,GAM,SOIDB
    7 FORMAT(5X,*PHI=*,F3.1,2X,*MUF=*,F4.1,2X,*GAMMA=*,F4.1,
     12X,*S/I=*,F5.1,/)
      PRINT 8
    8 FORMAT(10X,*WAT*,9X,*THETA*,9X,*E1*,11X,*E2*,11X,*E3*,
     111X,*E4*,9X,*MOD*,10X,*INT*)
      DO 40 JJ=1,21
       PRINT 9, WAT(JJ),THETA(JJ),E1(JJ),E2(JJ),E3(JJ),E4(JJ),YY(JJ),
     1YYI(JJ)
   40 CONTINUE
    9 FORMAT(5X,8(1PE12.5,1X))
      PRINT 10
   10 FORMAT(5X,*-------------------------------------------------------
     1-----------------------------------------------*)
      PRINT 11,SUM1,SUM3,SUM5,SUM7,SUM9,SUM11,SUM13
   11 FORMAT(18X,7(1PE12.5,1X))
      PRINT11,SUM2,SUM4,SUM6,SUM8,SUM10,SUM12,SUM14
      PRINT 12
```


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```
12   FORMAT(5X,*=============================================================
     1=============================================*,//)
60   CONTINUE
     PRINT 6
70   CONTINUE
80   CONTINUE
     END
```

PHI=0.0 MUF= 1.0 GAMMA= 1.5 S/I= 10.0

WAT	THETA	E1	E2	E3	E4	MOD	INT
0.	-1.00000E+00	6.32456E+00	-6.32456E+00	-5.00000E-01	-3.16228E+00	1.00000E+00	1.00000E+00
1.57080E-01	-9.81220E-01	5.43548E+00	-5.33340E+00	-5.09204E-01	-2.76777E+00	9.87688E-01	9.69714E-01
3.14159E-01	-9.65215E-01	3.55382E+00	-3.43020E+00	-5.16723E-01	-1.83634E+00	9.51057E-01	8.80690E-01
4.71239E-01	-1.02233E+00	2.22627E+00	-2.27598E+00	-4.88603E-01	-1.08776E+00	8.91007E-01	7.38320E-01
6.28319E-01	-7.90054E-01	2.22755E+00	-1.75989E+00	-5.68545E-01	-1.26646E+00	8.09017E-01	5.51228E-01
7.85398E-01	-5.61227E-01	2.95833E+00	-1.66029E+00	-5.10573E-01	-1.51044E+00	7.07107E-01	3.30748E-01
9.42478E-01	-5.68365E-01	3.32901E+00	-1.89210E+00	-5.14658E-01	-1.71330E+00	5.87785E-01	9.02328E-02
1.09956E+00	-7.99361E-01	3.00113E+00	-2.39898E+00	-5.67610E-01	-1.70347E+00	4.53991E-01	-1.55748E-01
1.25664E+00	-7.55717E-01	2.58151E+00	-1.95089E+00	-5.69852E-01	-1.47108E+00	3.09017E-01	-3.92294E-01
1.41372E+00	1.11661E-01	2.64420E+00	2.95255E-01	1.11644E-01	2.95209E-01	1.56434E-01	-6.05079E-01
1.57080E+00	4.68923E-01	3.01149E+00	1.41216E+00	4.47296E-01	1.34703E+00	1.79489E-09	-7.81212E-01
1.72788E+00	7.74434E-02	3.16982E+00	2.45482E-01	7.74406E-02	2.45473E-01	-1.56434E-01	-9.10025E-01
1.88496E+00	-4.14596E-01	2.99051E+00	-1.23985E+00	-4.02698E-01	-1.20427E+00	-3.09017E-01	-9.83716E-01
2.04204E+00	6.15271E-02	2.84451E+00	1.75015E-01	6.15262E-02	1.75012E-01	-4.53990E-01	-9.97821E-01
2.19911E+00	1.18254E+00	2.91579E+00	3.44802E+00	4.00114E-01	1.14665E+00	-5.87785E-01	-9.51485E-01
2.35619E+00	1.34304E+00	3.09574E+00	4.15769E+00	3.15749E-01	9.77476E-01	-7.07107E-01	-8.47516E-01
2.51327E+00	5.97971E-01	3.16228E+00	1.89095E+00	5.30184E-01	1.67659E+00	-8.09017E-01	-6.92210E-01
2.67035E+00	5.62019E-02	2.99902E+00	1.68550E-01	5.62013E-02	1.68549E-01	-8.91007E-01	-4.94976E-01
2.82743E+00	7.41741E-01	2.80586E+00	2.09123E+00	5.69388E-01	1.59763E+00	-9.51057E-01	-2.67760E-01
2.98451E+00	1.87805E+00	2.82599E+00	5.30735E+00	1.39733E-01	3.94884E-01	-9.87688E-01	-2.43245E-02
3.14159E+00	1.52362E+00	3.16228E+00	4.81811E+00	2.38477E-01	7.54129E-01	-1.00000E+00	2.20584E-01
	6.25669E-01	3.12609E+00	-1.75656E-01	-1.03498E-01	-3.86024E-01	1.14030E-09	-1.96647E-01
	6.85270E-01	1.03404E+01	7.59643E+00	1.59027E-01	1.80202E+00	4.96922E-01	4.70740E-01

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The purposes of this study (and subsequent efforts) are several: (1), to extend earlier models of the FM reception process, to include as much "realisim" - i.e., non-ideality of both the linear and nonlinear elements of the typical FM receiver - as possible, and still retain analytical and computational feasibility; (2), to examine explicit cases of interference produced by one or more deterministic signals; and (3), with such specific examples, both to provide insights into the distortion effects generated by the nonlinear interactions of the various (desired and undesired) signals in the receiver and to present the analytical framework of the instantaneous outputs required in any (subsequent) fully statistical treatment, where now the interference (e.g., "noise") is noticeably nongaussian. In addition, these deterministic models may also provide useful structures for simulation studies.

(Abstract continued on back)

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Abstract continued

The instantaneous receiver outputs are obtained for the following receiver models, (A), and interference "scenarios", (B): for (A): (I) "superclipping: and an ideal discriminator; (II), no limiting and ideal discriminator; III, "superclipping" and a nonideal discriminator; IV, no limiting and a nonideal discriminator. For (B), with each (A), we consider explicitly the cases of: (i), one cochannel interfering signal; (ii), one adjacent channel interferer, and (iii), M symmetrical interferers (M = 1, 5). Also included are the mean and mean-square outputs. All the above are obtained here for idealized (i.e. sufficiently wideband) RF-IF receiver stages, which are essentially linear under this condition. The results are illustrated with cases for selected, typical parameters of the combination of the interference-receiver structure. For other combinations, the appropriate computer programs are included in the Appendix.

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